

COMPUTER-AIDED OPTIMUM DESIGN  
OF A CLASS OF MIXED LUMPED-DISTRIBUTED  
BUTTERWORTH AND CHEBYSHEV FILTERS

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A Major Technical Report

in

The Faculty

of

Engineering

Presented in Partial Fulfillment of the  
Requirements for the Master of Engineering Degree at  
Concordia University  
Montreal, Canada  
April 1980

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ABSTRACT

Analytical investigation of the behavior of mixed lumped-distributed structures is found to be complicated, long and time consuming. Hence a computer-aided study of mixed lumped-distributed structures is carried out in this report.

The element values are taken from the filter tables (Ref. 3). Three different cases, in which the order of lumped elements are different, are considered both in low-pass and high-pass cases.

The low-pass and high-pass responses with Butterworth and Chebyshev element values are studied and the possibility of Butterworth and Chebyshev filter design is investigated.

While designing Chebyshev filters, it was found necessary to develop an algorithm which can contain the passband ripple within certain band. For reasons discussed in this report it was also necessary to develop an algorithm which should give 'n' parameter values for a transfer function having 'n' lumped elements.

The program 'AFTAB' is developed in this report and this will enable one to carry out the required design.

#### ACKNOWLEDGEMENT

The author wishes to express deep sense of gratitude to Professor V. Ramachandran for the suggestion of the topic, the guidance during the investigation of the problem and preparation of this report.

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# LIST OF ABBREVIATIONS

LP	Low-pass
HP	High-pass
Z	Characteristic impedance of transmission line
D	Group or time delay of the transmission line
s	Complex frequency variable
U.E.	Unit element
r	Ratio of terminating resistance
r/s	Radians per second
$\omega_r$	3 db Frequency
$\omega$	Frequency r/s
T.L.	Transmission Line
$V_r$	$\left  \frac{V_0}{V_s} \right $



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## CHAPTER 1

### INTRODUCTION

Filters are used for signal processing in communications. They are classified by the nature of their response. The most commonly used are low-pass, bandpass, high-pass, bandstop and all-pass filters.

In practice the design is carried out for a low-pass filter and by frequency transformation it can be converted to high-pass, bandpass or bandstop.

The filters also differ in frequency classification. They can have various types of response in the same frequency range, namely, Butterworth, Chebyshev and elliptic.

In recent years efforts have been made to mix the lumped filter with uniform lossless transmission line. The properties of uniform lossless transmission are well known.

We are investigating some of the mixed-lumped structures in this report. We also design low-pass and high-pass Butterworth and Chebyshev filters with mixed lumped-distributed structure.

#### 1.1 LUMPED-DISTRIBUTED NETWORKS

The driving point immittance functions for networks consisting of only lumped, linear, finite, passive, bilateral (LLFPB) elements is a rational function of a complex frequency variable  $s$ , and can be written as

$$Z(s) = \frac{P(s)}{Q(s)} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \dots + b_{m-1} s + b_m} \quad \dots(1)$$

This can also be represented as

$$Z(s) = \frac{a}{b} \frac{(s - z_1)(s - z_2) \dots (s - z_n)}{(s - p_1)(s - p_2) \dots (s - p_m)} \quad \dots(2)$$

Where  $z$ 's and  $p$ 's are the zeroes and poles of  $Z(s)$  respectively. The polynomials  $P(s)$  and  $Q(s)$  have roots only in the left half plane and hence they are Hurwitz polynomials.

The coefficients  $a_i$ 's and  $b_i$ 's ( $i = 0, 1, 2, \dots$ ) for that matter are positive real constants.

An immittance function is only realizable if it is a positive real function.

A function  $F(s)$  is positive real if and only if

- (a)  $F(s)$  is real when  $s$  is real
- (b)  $\operatorname{Re} F(s) \geq 0$  for  $\operatorname{Re} s \geq 0$

If the network consists of lumped distributed elements then its immittance functions need not be a rational function in  $s$ .

Networks containing lumped reactances in addition to lossless uniform transmission lines (UE's) are of the lumped-distributed type and the network functions are irrational such lumped-distributed networks are highly desirable because of the following advantages.



### 1.1.1 ADVANTAGES OF LUMPED-DISTRIBUTED NETWORKS

- (i) In the case of lumped-distributed networks, allowance may be made for the parasitics in the terminating impedances, that is, the terminations need not be purely resistive.
- (ii) A conventional Quarterwave transformer gives small or no attenuation at the higher harmonics, whereas the lumped-distributed impedance transformer can be designed to have the properties of an impedance transformer and a low-pass filter.
- (iii) In the case of a comb-line filters, lumped capacitive coupling at the input and output reduces the filter size by eliminating the transmission line matching section.
- (iv) In the case of cascaded U.E. filters, the number of U.E's can be reduced from  $(2n+1)$  to  $n$  if the cascaded lines are separated by  $(n+1)$  lumped capacitors.

### 1.2.1 ANALYSIS OF LUMPED-DISTRIBUTED STRUCTURE FOR $n = 4$

In this chapter we will consider the design of a LP filter as shown in Fig. 1.2.1.1. The lumped elements are the inductors and capacitors and the distributed network is the U.E. (transmission line) with characteristic impedance  $z = 1$  ohm. The entire network is doubly terminated in 1 ohm resistor. The numbering of the lumped elements are done in accordance with the filter tables.

To find the overall transfer function of the network shown in Fig. 1.2:1.1, chain matrix method is used.

The overall chain matrix of the network is the product

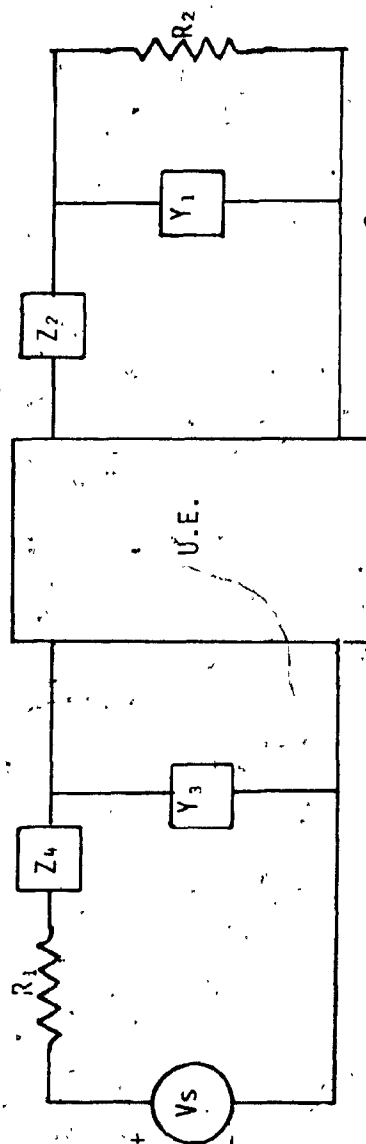


Fig. 1.2.1.1

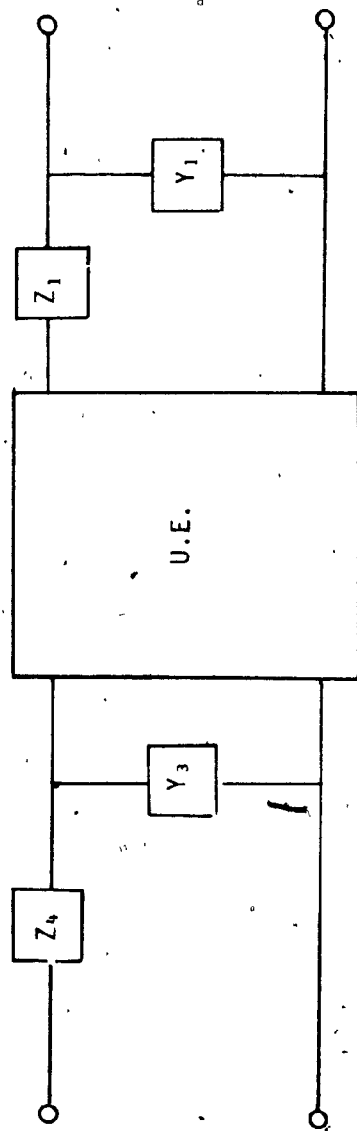


Fig. 1.2.1.2

of chain matrix of the individual elements.

We consider the network of Fig. 1.2.1.2. The chain matrix for the series element, which is an inductor in low-pass and capacitor in high-pass, is

$$\begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix}$$

and for the shunt element, a capacitor in low-pass and an inductor in high-pass, is

$$\begin{bmatrix} 1 & 0 \\ y & 1 \end{bmatrix}$$

For the transmission line (U.E.), the chain matrix is

$$\begin{bmatrix} \cosh(sd) & z \sinh(sd) \\ \frac{1}{z} \sinh(sd) & \cosh(sd) \end{bmatrix}$$

where

$z$  = characteristic impedance

$d$  = time delay of the transmission line

The overall chain matrix of the network is the product of the successive chain matrices. That is

$$\begin{bmatrix} 1 & z_4 & 1 & 0 \\ 0 & 1 & y_3 & 1 \end{bmatrix} \begin{bmatrix} \cosh(sd) & z \sinh(sd) \\ \frac{1}{z} \sinh(sd) & \cosh(sd) \end{bmatrix} \begin{bmatrix} 1 & z_2 & 1 & 0 \\ 0 & 1 & y_1 & 1 \end{bmatrix}$$

Hence the overall chain parameters A, B, C, D are

$$A = ((1 + z_4 y_3) \cosh(sd) + (\frac{z_4}{z} z \sinh(sd)(1 + z_2 y_1) + (((1 + z_4 y_3) z \sinh(sd) \dots \\ \dots + z_4 \cosh(sd)) y)$$

$$B = ((1 + y_3 z_4) \cosh(sd) + \frac{z_4}{z} \sinh(sd)) z_2 + (1 + y_3 z_4) z \sinh(sd) \dots \\ \dots + z_4 \cosh(sd)$$

$$C = (y_3 \cosh(sd) + \frac{1}{z} \sinh(sd)(1 + y_1 z_2) + (y_1 y_3 z \sinh(sd) + y_1 \cosh(sd)$$

$$D = z_2 y_3 \cosh(sd) + \frac{z_2}{z} \sinh(sd) + y_3 z \sinh(sd) + \cosh(sd)$$

### 1.2.2 ANALYSIS OF LUMPED-DISTRIBUTED STRUCTURE FOR $n = 2$

The overall chain matrix of the structure of Fig. 2.2 is the product of the chain matrix of the individual elements. That is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cosh(sd) & z \sinh(sd) \\ \frac{1}{z} \sinh(sd) & \cosh(sd) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ y_1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cosh(sd) + \frac{z_2}{z} \sinh(sd) & z \sinh(sd) + z_2 \cosh(sd) \\ \frac{1}{z} \sinh(sd) & \cosh(sd) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ y_1 & 1 \end{bmatrix}$$

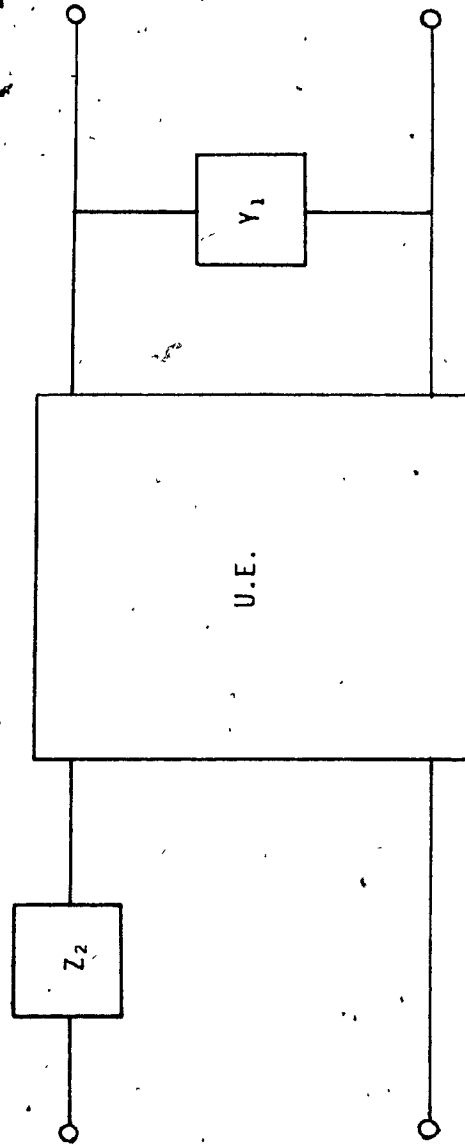


Fig. 1.2.2

$$= \begin{bmatrix} \cosh(sd) + \frac{z_2}{z} \sinh(sd) + y_1 z \sinh(sd) & z \sinh(sd) + z_2 \cosh(sd) \\ \frac{1}{z} \sinh(sd) + y_1 \cosh(sd) & \cosh(sd) \end{bmatrix}$$

Hence the chain parameter A, B, C, D are

$$A = (1 + y_1 z_2) \cosh(sd) + (z_2/z + y_1 z_2) \sinh(sd)$$

$$B = z \sinh(sd) + z_2 \cosh(sd)$$

$$C = \frac{1}{z} \sinh(sd) + y_1 \cosh(sd)$$

$$D = \cosh(sd)$$

### 1.2.3 ANALYSIS OF LUMPED-DISTRIBUTED STRUCTURE FOR $n = 6$

For the analysis of the structure of Fig. 1.2.3 we use the same procedure of multiplying the successive individual chain matrices

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z_6 & 1 & 0 \\ 0 & 1 & y_5 & 1 \end{bmatrix} \begin{bmatrix} 1 & z_4 & \cosh(sd) & z \sinh(sd) \\ 0 & 1 & \frac{1}{z} \sinh(sd) & \cosh(sd) \end{bmatrix} \dots$$

$$\dots \begin{bmatrix} 1 & 0 & 1 & z_2 & 1 & 0 \\ y_3 & 1 & 0 & 1 & y_1 & 1 \end{bmatrix}$$

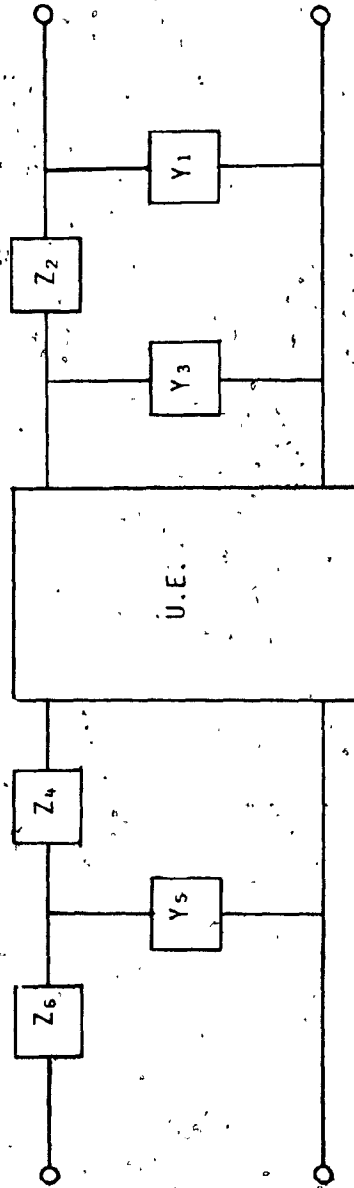


Fig. 1.2.3



The overall chain parameters are,

$$A = (1 + z_6 y_5) \cosh(sd) + ((1 + z_6 y_5) z_4 + z_6) \frac{1}{z} \sinh(sd) + ((1 + z_6 y_5) z \sinh(sd) \dots \\ \dots + ((1 + z_6 y_5) z_4 + z_6) \cosh(sd)) y_3 + (((1 + z_6 y_5) z_4 + z_6) \frac{1}{z} \sinh(sd) \dots \\ \dots + ((1 + z_6 y_5) z \sinh(sd) + ((1 + z_6 y_5) z_4 + z_6) \cosh(sd)) y_3) z_2 \dots \\ \dots + (1 + z_6 y_5) z \sinh(sd) + ((1 + z_6 y_5) z_4 + z_6) \cosh(sd)) y_1$$

$$B = -((1 + z_6 y_5) \cosh(sd) + ((1 + z_6 y_5) z_4 + z_6) \frac{1}{z} \sinh(sd) + ((1 + z_6 y_5) z \sinh(sd) \dots \\ \dots + ((1 + z_6 y_5) z_4 + z_6) \cosh(sd)) y_3) z_2 + (1 + z_6 y_5) z \sinh(sd) \dots \\ \dots + ((1 + z_6 y_5) z_4 + z_6) \cosh(sd)$$

$$C = y_5 \cosh(sd) + (1 + y_5 z_4) \frac{1}{z} \sinh(sd) + (y_5 z \sinh(sd) + (1 + y_5 z_4) \cosh(sd)) y_3 \dots \\ \dots + ((y_5 \cosh(sd) + (1 + y_5 z_4) \frac{1}{z} \sinh(sd) + (y_5 z \sinh(sd) \dots \\ \dots + (1 + y_5 z_4) \cosh(sd)) y_3) z_2 + y_5 z \sinh(sd) + (1 + y_5 z_4) \cosh(sd)) y_1$$

$$D = (y_5 \cosh(sd) + (1 + y_5 z_4) \frac{1}{z} \sinh(sd) + (y_5 z \sinh(sd) \dots \\ \dots + (1 + y_5 z_4) \cosh(sd)) y_3) z_2 + y_5 z \sinh(sd) + (1 + y_5 z_4) \cosh(sd)$$

### 1.3 DERIVATION OF TRANSFER FUNCTION FROM CHAIN MATRIX

The network of Fig. 1.2.1.1 can be represented for simplicity by Fig. 1.3. The chain matrix of the lumped-distributed network is known from previous derivations, and are represented as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_0 \\ -I_0 \end{bmatrix}$$

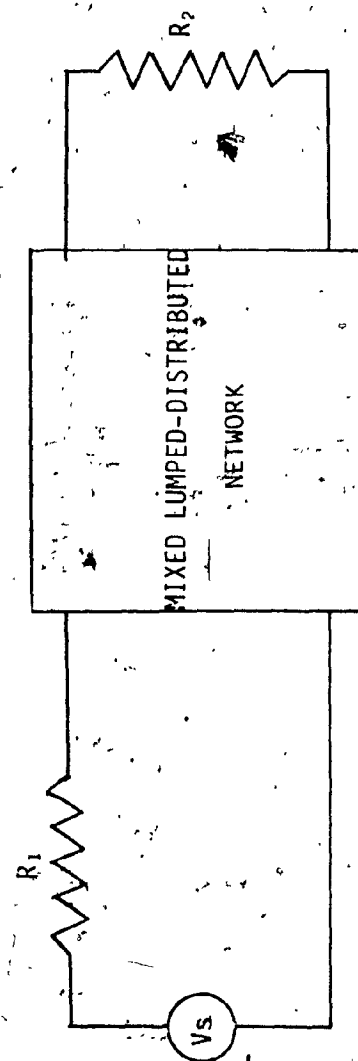


Fig. 1.3

$$v_i = Av_0 - BI_0 \quad \dots(1)$$

$$I_i = Cv_0 - DI_0 \quad \dots(2)$$

$$v_s = I_i R_1 + v_i \quad \dots(3)$$

$$-I = v_0/R_2 \quad \dots(4)$$

Substitution of equation (4) in equation (1) yields

$$v_i = (A + B/R_2)v_0 \quad \dots(5)$$

Similarly substitution of equation (4) in equation (2) yields

$$I_i = (C + D/R_2)v_0 \quad \dots(6)$$

Substituting equation (5) and equation (6) in equation (3) we get

$$v_s = (C + D/R_2)v_0 R_1 + (A + B/R_2)v_0 \quad \dots(7)$$

The transfer function

$$\frac{v_0}{v_s} = 1/\{(C + D/R_2)R_1 + (A + B/R_2)\} \quad \dots(8)$$

The terminating resistances  $R_1$  and  $R_2$  are equal to 1 ohm. Therefore the transfer function is

$$\frac{v_0}{v_s} = 1/(A + B + C + D) \quad \dots(9)$$

where A, B, C, D are obtained earlier.

#### 1.4 SCOPE OF THE REPORT

This report aims at the study of Mixed Lumped-Distributed network cascaded in a manner that the transmission line is preceded and followed by lumped networks of different order. The cascaded network is then doubly

terminated in 1 ohm resistors.

Analytical investigations of such structures have been found to be complicated, long and time consuming. These difficulties necessitated the use of computer programs to investigate the frequency domain behavior of such structures both in low-pass and high-pass cases.

An effort has been made to design low-pass and high-pass Butterworth and Chebyshev filters from these structures. To design the low-pass and high-pass Chebyshev filter we have developed an algorithm which minimizes the passband ripple.

The 3 db frequency in this report is termed as  $\omega_r$ . The vertical axis in all the computer plots represents the ratio  $|\frac{V_0}{V_s}|$  which is abbreviated as  $V_r$  in this report.

## CHAPTER 2

### LOW-PASS FILTERS

#### 2.1 GENERAL LOW-PASS STRUCTURES

The behavior of an ideal low-pass filter is shown in Fig. 2.1.1.

This ideal response shows no alternation at all from zero frequency to  $\omega = 1$ , but infinite alternation above cutoff frequency. Practical filters are approximated to give a response close to the ideal response.

For practical design procedures, to avoid the use of numbers which involve powers of 10 and  $2\pi$ , we begin with designs which are normalized for 1-ohm terminations and for cutoff frequency at one radian per second.

Such low values of frequency and impedance yield the impractical element values such as the capacitors in farads rather than more usual microfarads. But once the normalized network values are obtained, it is scaled in both impedance and frequency to yield a practical filter.

There are various types of standard low-pass responses close to the ideal response. Fig. 2.1.2 shows a Butterworth type of response. This type of response, besides being flat near zero frequency and giving high attenuation at high frequency, is not close enough to the ideal response in the vicinity of the cutoff.

The Chebyshev response of Fig. 2.1.3 gives ripple in the passband which is desired for power transmission.

The Chebyshev approximation is about equally good throughout the

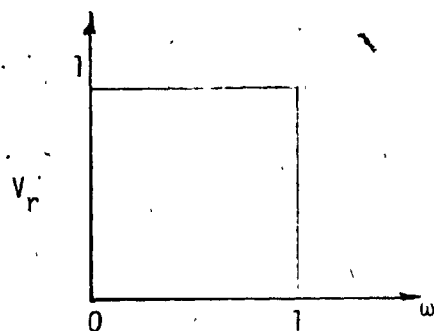


Fig. 2.1.1 Ideal low-pass response

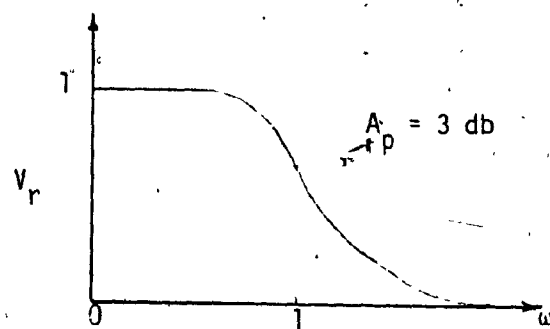


Fig. 2.1.2 Butterworth approximation to ideal response

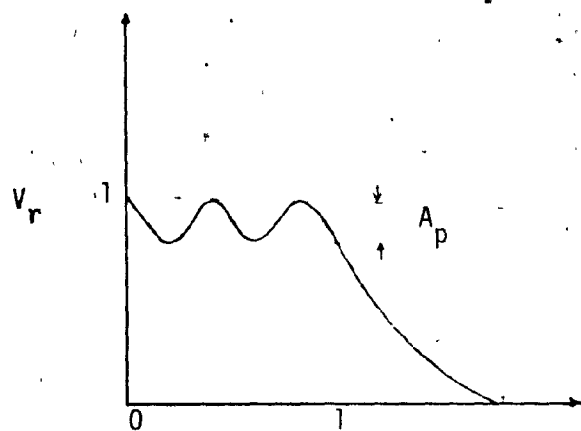


Fig. 2.1.3 Chebyshev approximation

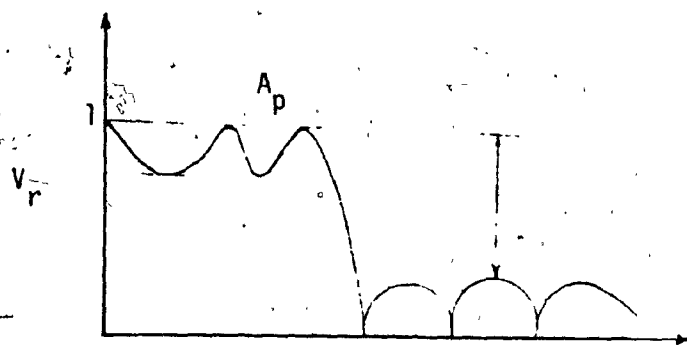


Fig. 2.1.4 Elliptic-function response

entire passband, but it can also be said to be about equally poor. Both of these types have zeroes of their responses only at infinite frequency. Given this condition it can be proved that the Chebyshev type gives the highest rate of attenuation into the stopband.

Even higher performances can be obtained if response zeroes are positioned near the passband. Fig. 2.1.4 shows elliptic function type of response having equal ripples in the passband and equally high return lobes in the stopband. This much attenuation throughout the stopband is usually required. For that reason elliptic filters are the most commonly used.

#### 2.2.1 THE MIXED LUMPED-DISTRIBUTED LOW-PASS STRUCTURES WITH FOUR LUMPED ELEMENTS

The structure considered is shown in Fig. 2.2.1. As can be seen, this is doubly terminated structure, consisting of four lumped elements and one distributed element. The chain parameters are determined in section 1.2.1 and they are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \{(1 + y_3 z_4) \cosh(sd) + \frac{z_4}{z} \sinh(sd) + (1 + y_3 z_4) y_1 z_2 \cosh(sd) + z_4 z_2 y_1 \sinh(sd)/z + y_1 (1 + y_3 z_4) z \sinh(sd) + z_4 y_1 \cosh(sd)\} & \{(1 + y_3 z_4) z_2 \cosh(sd) + z_4 z_2 \sinh(sd)/z + (1 + y_3 z_4) z \sinh(sd) + z_4 \cosh(sd)\} \\ \{y_3 \cosh(sd) + \frac{1}{z} \sinh(sd) + y_3 y_1 z_2 \cosh(sd) + z_2 y_1 / z \sinh(sd) + y_3 y_1 \sinh(sd) + y_1 \cosh(sd)\} & \{y_3 z_2 \cosh(sd) + \frac{z_2}{z} \sinh(sd) + y_3 z \sinh(sd) + \cosh(sd)\} \end{bmatrix}$$

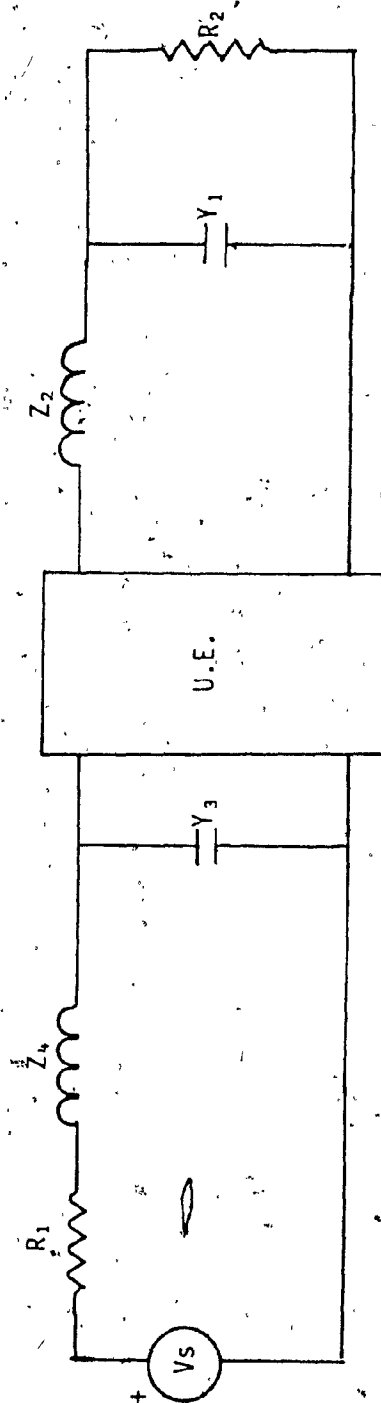


Fig. 2.2.1



PROGRAM LPN4

73/174 OPT=1

FTN 4.8+498

80/03/05.

```

1      PROGRAM LPN4(INPUT,OUTPUT)
      C   THIS PROGRAM PLOTS THE LOSS CHARACTERISTICS OF
      C   THE GIVEN TRANSFER FUNCTION USING THE LIBRARY
5      C   SUBROUTINE 'USPLH'.
      C   THE ELEMENT VALUES ARE TAKEN FROM THE AVAILABLE
      C   FILTER TABLES FOR NORMALIZED CHEBYSHEV FUNCTION
      C   WITH 1 DB, RIPPLE AND NORMALIZED BUTTERWORTH FUNCTIONS.

10     COMPLEX S,HS1,HS2,HS3,HS4,HS5,ZS,ZC,Y1,Z2,Y3,Z4
      DIMENSION X(201),Y(201,1),A(160),IMAG4(5151)
      REAL C1,L2,C3,L4,
      READ*,C1,L2,C3,L4,D,Z
      PRINT 11
15     11  FORMAT(1H1,3(/))
      PRINT 12
      12  FORMAT(15X,*ELEMENT VALUES*)
      PRINT 13
      13  FORMAT(15X,14(1H-))
20     PRINT 14,C1,L2
      PRINT 15,C3,L4
      PRINT 16,D,Z
      14  FORMAT(15X,*C1=*F7.4,3X,*L2=*F7.4)
      15  FORMAT(15X,*C3=*F7.4,3X,*L4=*F7.4)
25     16  FORMAT(15X,*D=*F5.3,2X,*Z=*F5.3)
      PRINT 18
      18  FORMAT(/,15X,25(1H-),/)
      PRINT 27
      PRINT 22
30     22  FORMAT(15X,**,* FREQ(R/8)*,3X,**,*6X,*ATTENUATION (DB)*,*,*)
      27  FORMAT(15X,39(1H*))
      PRINT 27
      DO 10 I=1,201,5
      W=(I-1)*0.01
35     S=CMPLX(0.0,W)
      ZS=CMPLX(0.0,SIN(W*D))
      ZC=CMPLX(COS(W*D),0.0)
      Y1=C1*S
      Z2=L2*S
40     Y3=C3*S
      Z4=L4*S
      HS1=((((1+Z4*Y3)*ZC+(Z4*ZS/Z))*((1+Z2*Y1)+
      *(((1+Z4*Y3)*Z*ZS+Z4*ZC)*Y1))
      HS2=((((1+Z4*Y3)*ZC+Z4*ZS/Z)*Z2+
45     *(((1+Z4*Y3)*Z*ZS+Z4*ZC))
      HS3=((Y3*ZC+ZS/Z)*((1+Z2*Y1))+
      *(Z*ZS*Y3*Y1+Y1*ZC))
      HS4=((Z2*Y3*ZC+Z2*ZS/Z))+
      *(Z*ZS*Y3+ZC))
      HS5=1/(HS1+HS2+HS3+HS4)
50     F1=CABS(HS5)
      Y(I,1)=20*ALOG10(F1)
      X(I)=W
      PRINT 33,W,Y(I,1)
55     33  FORMAT(15X,**,2X,F5.2,6X,**,3X,F14.8,4X,**)
      10  CONTINUE
      PRINT 27
      PROGRAM LPN4      73/174 OPT=1      FTN 4.8+498      80/03/05.

      READ 19,(A(I),I=1,160)
      19  FORMAT(80A1)
      CALL USPLH(X,Y,201,1,1,201,A,IMAG4,IER)
60     STOP
      END

```

As shown earlier, it is not easy to design the above filter by analytical means. Hence, computer-aided analysis and design appears to be a method for the solution. Before proceeding with the design, we should first obtain its frequency response, as this will enable us to formulate the problem of design. For this purpose we have to obtain the low-pass frequency responses with the case in which lumped elements constitute a Butterworth filter and with the case where lumped elements constitute a Chebyshev filter with a desired ripple.

#### 2.2.1.1 LP FILTERS WITH BUTTERWORTH ELEMENT VALUES

To obtain the low-pass response of the structure of Fig. 2.2.1 in which the lumped elements constitute a Butterworth filter, we substituted the Butterworth element values for  $n = 4$  and  $r = 1$  from the filter tables (Ref. 3) where  $n$  represents the order of the lumped network and  $r$  is the ratio of the terminating resistance which in our case is 1 as both the terminating resistances in our case are 1 ohm.

The computer plots of Fig. p1.1 to Fig. p1.6 give the low-pass frequency responses of the structure of Fig. 2.2.1 with Butterworth element values. The responses are plotted against increasing value of the Transmission Line time delay 'D' starting from zero.

As can be seen from Fig. p1.1, which is the low-pass frequency response of the structure of Fig. 2.2.1 when 'D' is zero, the 3 db frequency  $\omega_p$  is 1 r/s and the magnitude of attenuation is about 30 dbs.

We increment the value of time delay 'D' by 0.2 in every step and observe its effect on  $\omega_p$  and the attenuation.

LP RESPONSE WITH BUTTERWORTH ELEMENT VALUES FOR $\eta=1$ AND $\theta=0.0$									
0.	1	1	1	1	1	1	1	1	1
-.3012E+01	1	1	1	1	1	1	1	1	1
-.6024E+01	1	1	1	1	1	1	1	1	1
-.9036E+01	1	1	1	1	1	1	1	1	1
-.1205E+02	1	1	1	1	1	1	1	1	1
-.1506E+02	1	1	1	1	1	1	1	1	1
-.1807E+02	1	1	1	1	1	1	1	1	1
-.2108E+02	1	1	1	1	1	1	1	1	1
-.2410E+02	1	1	1	1	1	1	1	1	1
-.2711E+02	1	1	1	1	1	1	1	1	1
-.3012E+02	1	1	1	1	1	1	1	1	1
0.	1	1	1	1	1	1	1	1	1

Fig. p1.1

 $\omega_p = 1 \text{ r/s}$ 

.2000E+01

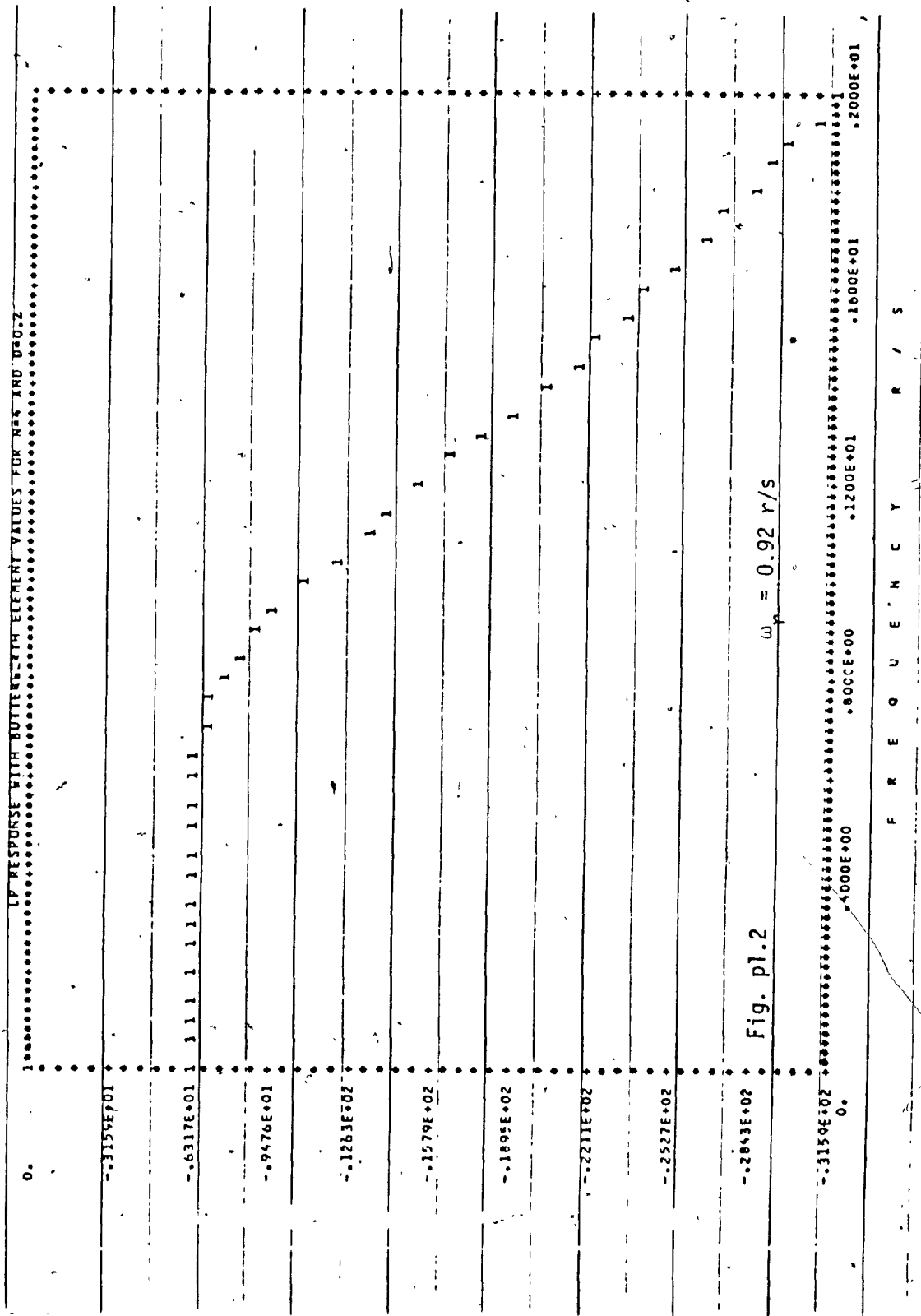
.1600E+01

.1200E+01

.8000E+00

.4000E+00

F R E Q U E N C Y R / S



IF RESPONSE WITH BUTTERWORTH ELEMENT VALUES FOR  $N=4$  AND  $0.0.6$

Q

**FOIA b 7 -**

-63148-01

**-0.9471E+01**

1263E-02

- . 137NF + 02

- .1094E+02

**2013-2014**

**-0.2925E+02**

- .2841E+02

Fig. p1:3

$$\omega_1 = 0.84 \text{ r/s}$$

5.3157402  
0.

○

4000E+00

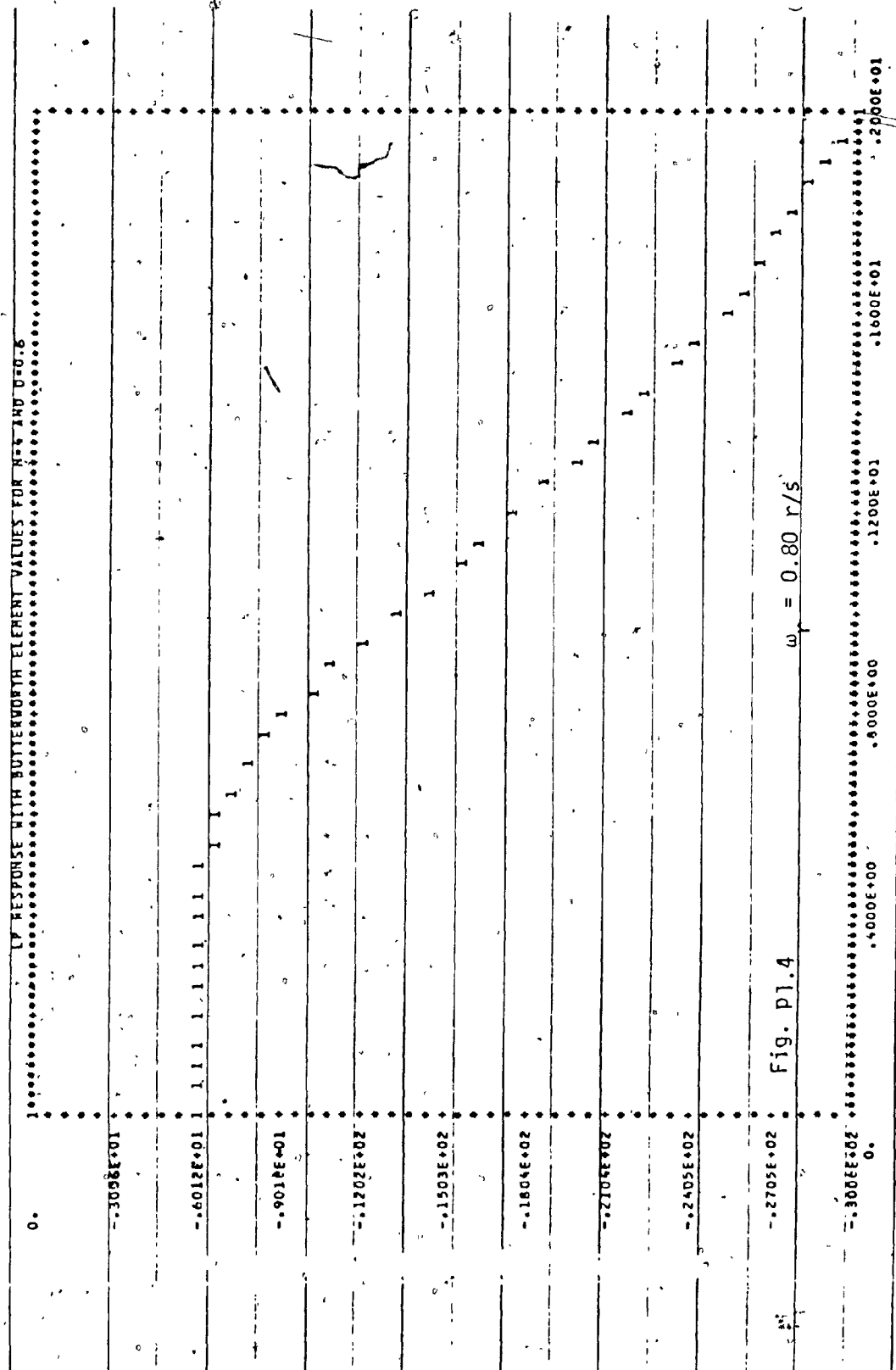
00+30000

1003021

10-30091-

**10-2000E-01**

SECRET



RESPONSE WITH BUTTERWORTH ELEMENT VALUES FOR  $N=5$  AND  $D=0.8$

0.

10-19692-

**-0.5272E+01**

-7909E+01

204502

-01310E+02

-01502E+02-

**201355070 -**

-02109E+02-

✓-2373E+03

Fig. p1.5

$$\omega = 0,76 \text{ r/s}$$

10.39592

50

004300050

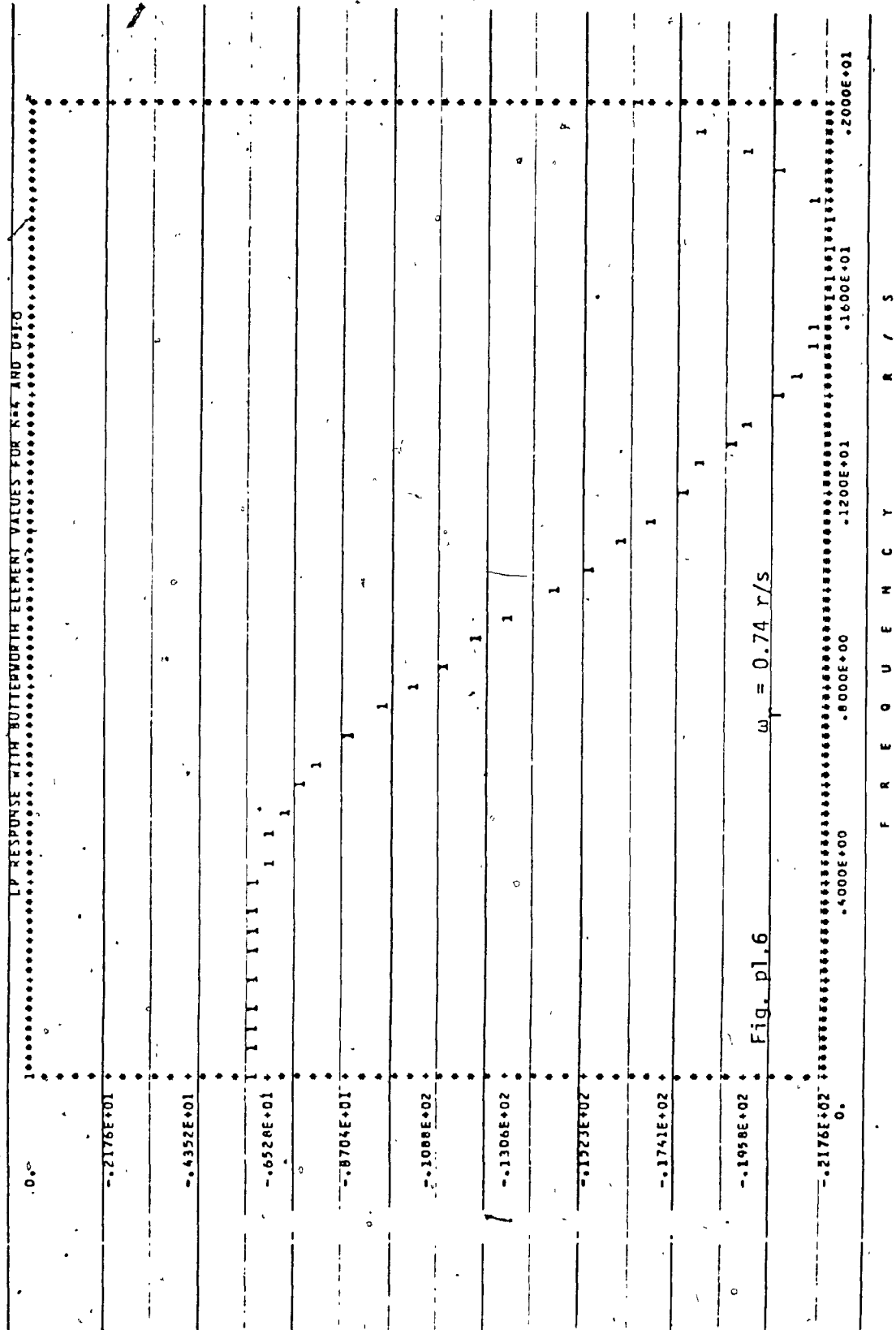
00-3000  
9999999999

10-30021  
1200E-01

• 1600E+01

10-30002

REFUGEE CYRUS, R/S



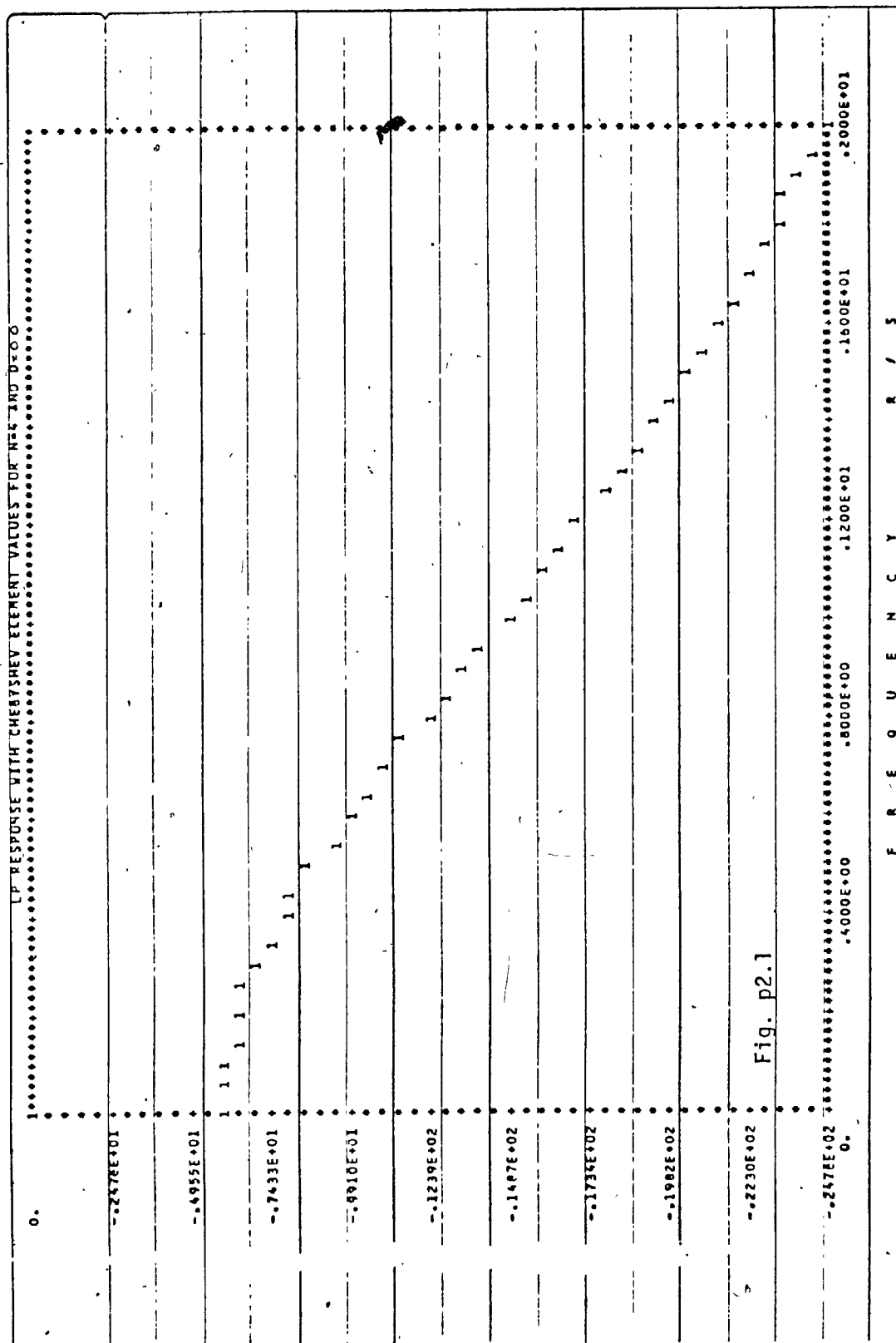


For  $D = 0.2$  the  $\omega_p$  is 0.92 r/s and the magnitude of attenuation is 32 dbs.  
 For  $D = 0.4$  the  $\omega_p$  is 0.84 r/s and the magnitude of attenuation is 32 dbs.  
 For  $D = 0.6$  the  $\omega_p$  is 0.80 r/s and the magnitude of attenuation is 30 dbs.  
 For  $D = 0.8$  the  $\omega_p$  is 0.76 r/s and the magnitude of attenuation is 26 dbs.  
 For  $D = 1.0$  the  $\omega_p$  is 0.74 r/s and the magnitude of attenuation is 16 dbs.  
 These results are derived from the computer plots of Fig. p1.1 to Fig. p1.6.  
 As we can see in Fig. p1.5 and Fig. p1.6 the frequency response is not maximally flat.

#### 2.2.1.2 LP FILTERS WITH CHEBYSHEV ELEMENT VALUES.

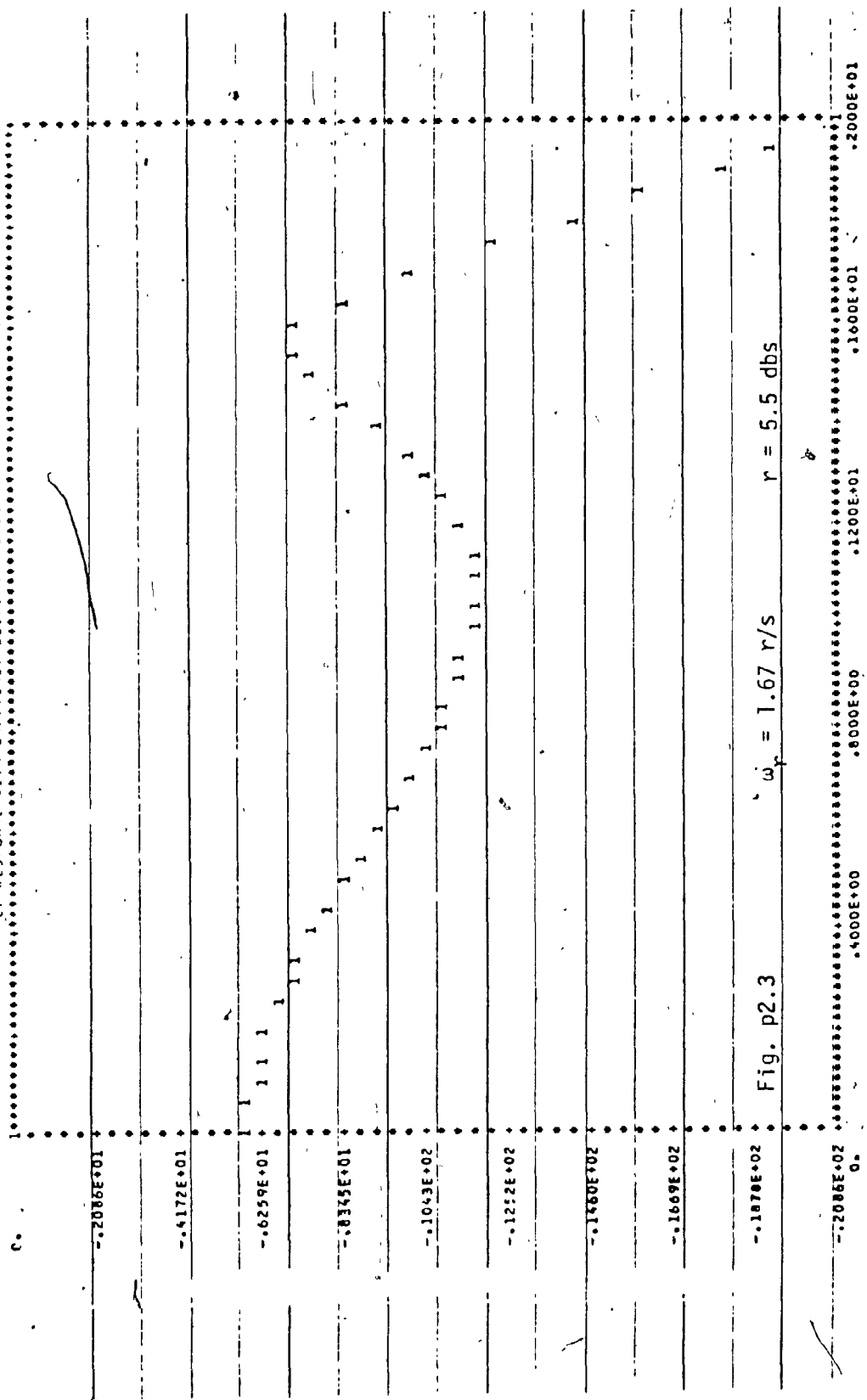
As in the case of Butterworth filters where we substituted Butterworth element values for the lumped elements, here also we have to substitute Chebyshev element values for the lumped elements in the structure of Fig. 2.2.1. But the filter table does not give Chebyshev element value for  $n = 4$  and  $r = 1$ , that is the number and lumped elements is even and the ratio of terminating resistance is unity. The filter table only gives odd order Chebyshev element values when  $r = 1$ . For that reason we chose the element values corresponding to  $n = 3$  and the passband ripple as 1 db. Here we had to suppose that one of the element values in our case, the inductance  $L_2$  is zero. This supposition of  $L_2$  to zero reduces the order of the network. But later a program 'AFTAB' is developed to take care of this problem and also to give the user a control over the passband ripple. This algorithm and its results are discussed later.

The computer plots of Fig. p2.1 to Fig. p2.6 are the low-pass response of the structure under discussion.





LP RESPONSE WITH CHEBYSHEV ELEMENT VALUES FOR  $N=4$  AND  $\Omega=0.4$



F R E Q U E N C Y      R / S

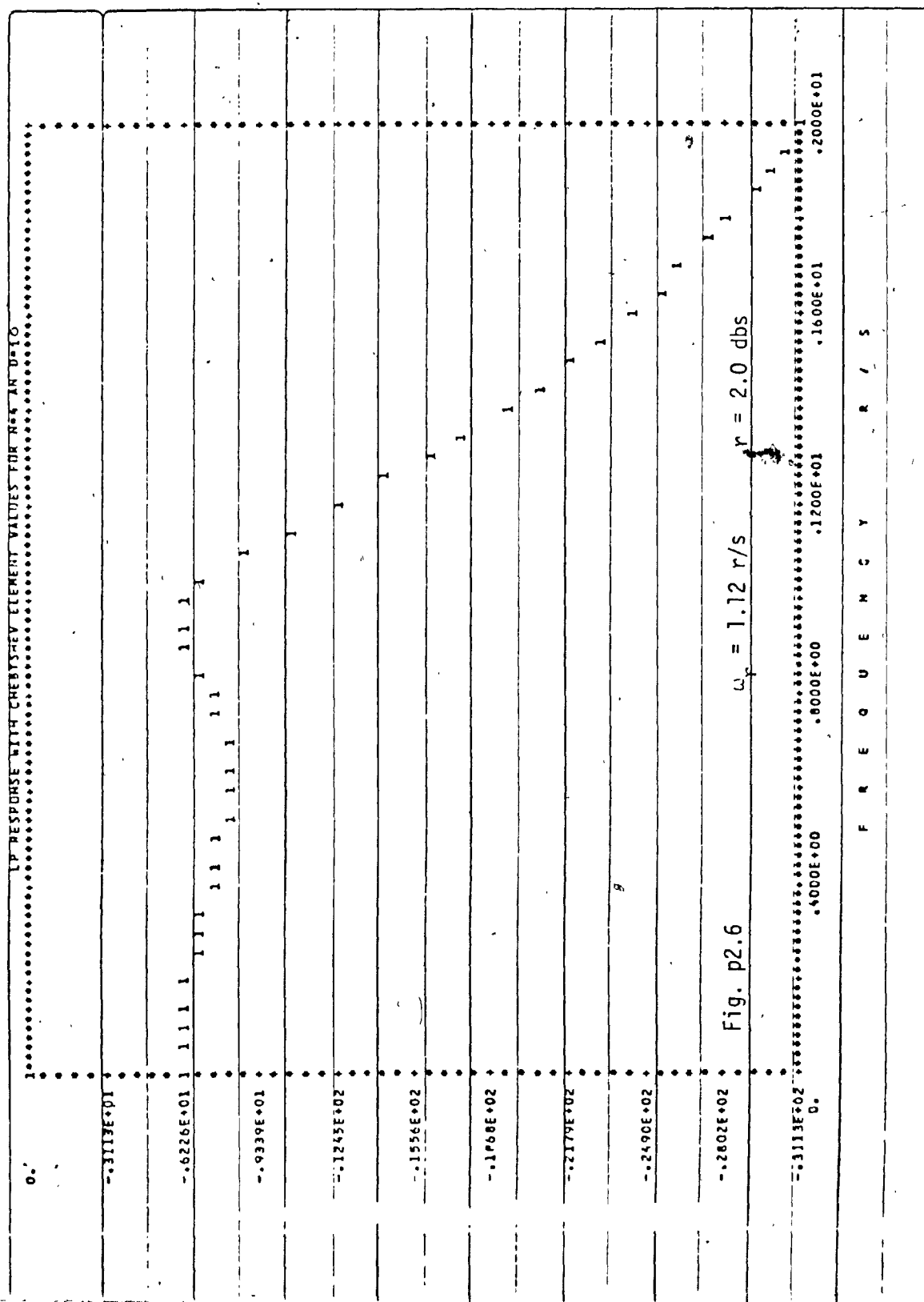
[illegible]

LP RESPONSE WITH CHEBYSHEV ELEMENT VALUES FOR $N=4$ AND $\Omega=0.8$									
0.	1	1	1	1	1	1	1	1	1
-.3017E+01	1	1	1	1	1	1	1	1	1
-.6034E+01	1	1	1	1	1	1	1	1	1
-.9051E+01	1	1	1	1	1	1	1	1	1
-.1207E+02	1	1	1	1	1	1	1	1	1
-.1508E+02	1	1	1	1	1	1	1	1	1
-.1810E+02	1	1	1	1	1	1	1	1	1
-.2112E+02	1	1	1	1	1	1	1	1	1
-.2413E+02	1	1	1	1	1	1	1	1	1
-.2715E+02	1	1	1	1	1	1	1	1	1
-.3017E+02	1	1	1	1	1	1	1	1	1
0.	1	1	1	1	1	1	1	1	1
-.4000E+00	1	1	1	1	1	1	1	1	1
-.8000E+00	1	1	1	1	1	1	1	1	1
-.1200E+01	1	1	1	1	1	1	1	1	1
-.1600E+01	1	1	1	1	1	1	1	1	1
-.2000E+01	1	1	1	1	1	1	1	1	1

Fig. p2.5

 $\omega_p = 1.24$  r/s $r = 2.6$  db

F R E Q U E N C Y R / S



As seen in the computer plots of Fig. p2.1 and p2.2, which are frequency response with Chebyshev element values for  $D = 0.0$  and  $D = 0.2$ , there is no passband ripple. Therefore it is not possible to design a Mixed Lumped-Distributed Chebyshev filter.

For  $D = 0.4$   $\omega_p$  is 1.67 r/s, the magnitude of attenuation is 21 db and passband ripple  $r = 5.5$  db.

For  $D = 0.6$   $\omega_p$  is 1.40 r/s, the magnitude of attenuation is 27 db and  $r = 3.7$  db.

For  $D = 0.8$   $\omega_p$  is 1.24 r/s, the magnitude of attenuation is 30 db and  $r = 2.6$  db.

For  $D = 1.0$   $\omega_p$  is 1.12 r/s, the magnitude of attenuation is 31 db and  $r = 2$  db.

#### 2.2.2.1 BUTTERWORTH FILTER DESIGN

As we know the cutoff frequency in Butterworth case has always to be 1. For  $D = 0$  we get this value. Thus the structure of Fig. 2.2.1 with Butterworth element values and zero group delay is a Mixed Lumped-Distributed filter for  $n = 4$  and zero time delay of the transmission line. In the cases where  $D$  is 0.8 and 1.0, it is not possible to design the Butterworth filter, as the response shown in Fig. p1.5 and p1.6 is not maximally flat. For the cases where  $D$  is 0.2, 0.4 and 0.6 we can design a Mixed Lumped-Distributed Butterworth filter by normalizing the cutoff frequency to 1 r/s. The corresponding scaling of the elements will then be carried out and those element will contribute a Butterworth filter for that particular value of  $D$ .



### 2.2.2.2 Chebyshev Filter Design

For the frequency response plots of Fig. p2.3 to Fig. p2.6, which are the response with Chebyshev element values we observe that the passband ripple has a value of 5.5 db for  $D = 0.4$  and decreases to 2 db for  $D = 1.0$ . The Chebyshev element values were chosen for a ripple of 1 db but in the Frequency response we are getting a ripple of over 2 db because of the presence of Transmission Line. Furthermore we have assumed one of the element values as zero for reasons discussed in section 2.2.1.2. Now we have to design an algorithm which will reduce the passband ripple and also give the values for all the four elements. Such an algorithm is discussed in next section.

### 2.2.3 Ripple Minimization and Chebyshev Filter Design

As discussed in the section 2.2.1.2 and section 2.2.2.2, we assumed that the value of one of the elements of the structure of Fig. 2.2.1 is zero. The value of passband ripple obtained from the low-pass response of the structure under discussion with Chebyshev element values, is above 5 db for  $D = 0.4$  and has a minimum value of 2 db at  $D = 1.0$ .

It was therefore necessary to develop an algorithm which could minimize the passband ripple and also give the values for all the four elements.

The program 'AFTAB' which employs the library subroutine 'ZXMIN' achieves this purpose.

The subroutine ZXMIN minimizes any specific error. We

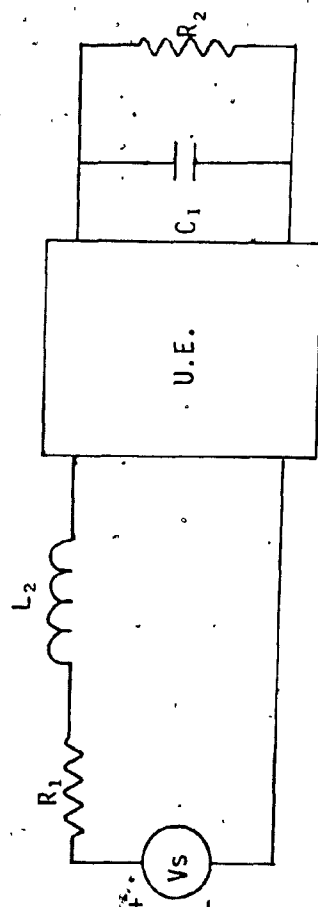


Fig. 2.3.1

PROGRAM AFTAB

73/174 OPT=1

FTN 4.0+498

80/03/05

```

1      PROGRAM AFTAB(INPUT,OUTPUT)
      C      THIS ALGORITHM IS DEVELOPED TO REDUCE THE PASS-BAND
      C      RIPPLE AND GIVE ALL THE PARAMETER VALUES OF THE
5      C      NETWORK. THE INPUT TO THIS ALGORITHM IS A SET OF
      C      PARAMETER VALUES WHICH YIELD A PASS-BAND RIPPLE OF
      C      ABOUT 4 DBS.
      C      THIS ALGORITHM IN GENERAL MINIMIZES THE PASS-BAND
      C      RIPPLE IN EVERY ITERATION AND PRINTS THE CORRESPONDING
10     C      VALUES OF THE PARAMETERS.
      C      THIS ALGORITHM USES THE LIBRARY SUBROUTINE 'ZXMIN'
      C      WHICH MINIMIZES ANY SPECIFIED ERROR, WHICH IN OUR CASE
      C      IS THE PASS-BAND RIPPLE.
15     DIMENSION X(6),G(6),H(21),W(18)
      C      EXTERNAL FUNCT.
      C      READ*,(X(I),I=1,6)
      C      PRINT 11
20     11  FORMAT(1H,5(/))
      C      PRINT 12
      C      12  FORMAT(20X,'INITIAL VALUES OF PARAMETERS')
      C      PRINT 14
      C      14  FORMAT(20X,30(1H*))
      C      PRINT 15,X(4)
25     15  FORMAT(20X,*C1=*,F8.6)
      C      PRINT 16,X(2)
      C      16  FORMAT(20X,*L2=*,F8.6)
      C      PRINT 17,X(3)
      C      17  FORMAT(20X,*C3=*,F8.6)
30     18  FORMAT(20X,*L4=*,F8.6)
      C      PRINT 19,X(5)
      C      19  FORMAT(20X,*D=*,F8.6)
      C      PRINT 20,X(6)
35     20  FORMAT(20X,*Z=*,F8.6)
      C      IOPT=0
      C      N=6
      C      MAXFN=50
      C      NSIG=4
40     PRINT 7
      C      PRINT 6
      C      6  FORMAT(20X,*PARAMETER VALUES AFTER EVERY ITERATION*)
      C      PRINT 7
      C      7  FORMAT(/,20X,40(1H*))
45     CALL ZXMIN(FUNCT,N,NSIG,MAXFN,IOPT,X,H,G,F,W,IER)
      C      STOP
      C      END

```

SUBROUTINE FUNCT

73/174 OPT=1

FTN 4.8+498

80/03/05.

```

1      SUBROUTINE FUNCT(N,X,F)
      COMPLEX S,HS1,HS2,HS3,HS4,HS5,Z8,ZC,Y1,Z2,Y3,Z4
      DIMENSION X(6)
5      REAL F,L1,L2,C1,C2,D,Z
      N=6
      L1=X(1)
      L2=X(2)
      C1=X(3)
10     C2=X(4)
      D=X(5)
      Z=X(6)
      FMAX=-6.5
      FMIN=-6.6
15     DO 50 I=1,100
      AM=(I-1)*0.01
      Y=AM*D
      S=CMPLX(0.0,AM)
      ZS=CMPLX(0.0,SIN(Y))
20     ZC=CMPLX(COS(Y),0.0)

      HS1=((((1+L1*C1*S**2)*ZC+(S*L1*ZS/Z))*((1+L2*C2*S**2)+
      *(((1+L1*C1*S**2)*Z*ZS+L1*S*ZC)*C2*S))
      HS2=((((1+L1*C1*S**2)*ZC+L1*S*ZS/Z)*L2*S+
25     *(((1+L1*C1*S**2)*Z*ZS+L1*S*ZC))*
      HS3=((((C1*S*ZC+ZS/Z)*((1+L2*C2*S**2))+
      *(Z*ZS*C1*C2*S**2+C2*S*ZC))
      HS4=((((L2*C1*ZC*S**2+L2*S*ZS/Z))+
30     *(Z*ZS*C1*S*ZC))

      HS5=1/(HS1+HS2+HS3+HS4)
      FF=CABS(HS5)
      FA=20*ALOG10(FF)
      IF(FA.LT.FMIN) FMIN=FA
      IF(FA.GT.FMAX) FMAX=FA
35     IF(FMIN.EQ.FMAX)
50     CONTINUE
      F=FMAX-FMIN
      PRINT 23,F
      23  FORMAT(20X,' RIPPLE=',F15.12)
40     PRINT 24,X(4),X(2)
      PRINT 25,X(3),X(1)
      PRINT 26,X(5),X(6)
      24  FORMAT(20X,'C1=',F15.12,4X,'L2=',F15.12)
      25  FORMAT(20X,'C3=',F15.12,4X,'L4=',F15.12)
45     26  FORMAT(20X,'D=',F15.12,4X,'D=',F15.12)
      PRINT 7
      7   FORMAT(/,20X,40(1H=),/)
      RETURN
      END

```

## INITIAL VALUES OF PARAMETERS

\*\*\*\*\*

C1=2.023600

L2=0.000000

C3=2.023600

L4=.994100

D=.800000

Z=1.000000

## =====

## PARAMETER VALUES AFTER EVERY ITERATION

=====

RIPPLE= 3.377141905018

C1= 2.023600000000 L2= 0.000000000000

C3= 2.023600000000 L4= .994100000000

D= .800000000000 Z= 1.000000000000

=====

RIPPLE= 3.377142578720

C1= 2.023600170577 L2= 0.000000000000

C3= 2.023600000000 L4= .994100000000

D= .800000000000 Z= 1.000000000000

=====

RIPPLE= 3.377142093010

C1= 2.023600000000 L2= .000000008429

C3= 2.023600000000 L4= .994100000000

D= .800000000000 Z= 1.000000000000

=====

RIPPLE= 3.377143181197

C1= 2.023600000000 L2= 0.000000000000

C3= 2.023600170577 L4= .994100000000

D= .800000000000 Z= 1.000000000000

=====

RIPPLE= 3.377142171044

C1= 2.023600000000 L2= 0.000000000000

C3= 2.023600000000 L4= .994100083796

D= .800000000000 Z= 1.000000000000

=====

=====

RIPPLE= 2.304607440540  
 C1= 1.924126645686 L2= .023843969015  
 C3= 1.923921484293 L4= .818990244871  
 D= .718480722183 Z= 1.031039690539

=====

RIPPLE= 1.760445075149  
 C1= 1.829590235662 L2= .075565424669  
 C3= 1.833594913557 L4= .647848826878  
 D= .669775886907 Z= 1.084942600758

=====

RIPPLE= 1.108432083193  
 C1= 1.640517415612 L2= .179008334377  
 C3= 1.652941772085 L4= .305565990891  
 D= .572366216353 Z= 1.192748421196

=====

RIPPLE= 1.108432162678  
 C1= 1.640517553897 L2= .179008334377  
 C3= 1.652941772085 L4= .305565990891  
 D= .572366216353 Z= 1.192748421196

=====

RIPPLE= 1.108432048457  
 C1= 1.640517415612 L2= .179008349467  
 C3= 1.652941772085 L4= .305565990891  
 D= .572366216353 Z= 1.192748421196

=====

RIPPLE= 1.108432142813  
 C1= 1.640517415612 L2= .179008334377  
 C3= 1.652941911418 L4= .305565990891  
 D= .572366216353 Z= 1.192748421196

=====

RIPPLE= 1.108432084685  
 C1= 1.640517415612 L2= .179008334377  
 C3= 1.652941772085 L4= .305566016649  
 D= .572366216353 Z= 1.192748421196

=====

=====

RIPPLE= .296504396593  
 C1= .792105031156 L2= .559684879356  
 C3= .621096044130 L4= .296274955329  
 D= .646605000460 Z= 1.095133622005

=====

RIPPLE= .296504497531  
 C1= .792105031156 L2= .559684879356  
 C3= .621096044130 L4= .296274930355  
 D= .646605054965 Z= 1.095133622005

=====

RIPPLE= .296504560365  
 C1= .792105031156 L2= .559684879356  
 C3= .621096044130 L4= .296274930355  
 D= .646605000460 Z= 1.095133714317

=====

RIPPLE= 1.639551582825  
 C1= -.851173000071 L2= .614032221236  
 C3= .008385431745 L4= .717579659343  
 D= .910691354472 Z= 1.087465667953

=====

RIPPLE= .246923998540  
 C1= -.029533984457 L2= .586858550296  
 C3= .314740737938 L4= .506927294849  
 D= .778648177466 Z= 1.091299644979

=====

RIPPLE= .246923988237  
 C1= -.029533976028 L2= .586858550296  
 C3= .314740737938 L4= .506927294849  
 D= .778648177466 Z= 1.091299644979

=====

RIPPLE= .246924057729  
 C1= -.029533984457 L2= .586858599765  
 C3= .314740737938 L4= .506927294849  
 D= .778648177466 Z= 1.091299644979

=====

=====

RIPPLE= .246924027803

C1= -.029533984457 L2= .586858550296  
C3= .314740737938 L4= .506927294849  
D= .778648177466 Z= 1.091299644979

=====

RIPPLE= .246923945816

C1= -.029533984457 L2= .586858550296  
C3= .314740737938 L4= .506927337579  
D= .778648177466 Z= 1.091299644979

=====

RIPPLE= .246924016035

C1= -.029533984457 L2= .586858550296  
C3= .314740737938 L4= .506927294849  
D= .778648243101 Z= 1.091299644979

=====

RIPPLE= .246924027524

C1= -.029533984457 L2= .586858550296  
C3= .314740737938 L4= .506927294849  
D= .778648177466 Z= 1.091299736969

=====

RIPPLE= .285787902729

C1= -.044420748559 L2= .563825096578  
C3= .264030648244 L4= .543665681900  
D= .795987361864 Z= 1.113020413317

=====

RIPPLE= .258162851970

C1= -.036977366508 L2= .575341823437  
C3= .289385693091 L4= .525296488374  
D= .787317769665 Z= 1.102160029148

=====

RIPPLE= .242010519736

C1= -.030278322663 L2= .585706877610  
C3= .312205233453 L4= .508764214201  
D= .779515136686 Z= 1.092385683396

=====



=====

RIPPLE= .242010509551  
 C1= -.030278314233 L2= .585706877610  
 C3= .312205233453 L4= .508764214201  
 D= .779515136686 Z= 1.092385683396

=====

RIPPLE= .242010578067  
 C1= -.030278322663 L2= .585706926982  
 C3= .312205233453 L4= .508764214201  
 D= .779515136686 Z= 1.092385683396

=====

RIPPLE= .242010548064  
 C1= -.030278322663 L2= .585706877610  
 C3= .312205259770 L4= .508764214201  
 D= .779515136686 Z= 1.092385683396

=====

RIPPLE= .242010467219  
 C1= -.030278322663 L2= .585706877610  
 C3= .312205233453 L4= .508764257087  
 D= .779515136686 Z= 1.092385683396

=====

RIPPLE= .242010536159  
 C1= -.030278322663 L2= .585706877610  
 C3= .312205233453 L4= .508764214201  
 D= .779515202394 Z= 1.092385683396

=====

RIPPLE= .242010549794  
 C1= -.030278322663 L2= .585706877610  
 C3= .312205233453 L4= .508764214201  
 D= .779515136686 Z= 1.092385775477

=====

TERMINATED

(IER = 131) FROM IMSL ROUTINE ZXMIN

have defined the passband ripple as the error function.

The algorithm minimizes the ripple in every iteration and prints the values of the ripple and all the four elements.

The ripple in this algorithm has been defined as the difference between the maximum of the maximas and the minimum of the minimas within the passband.

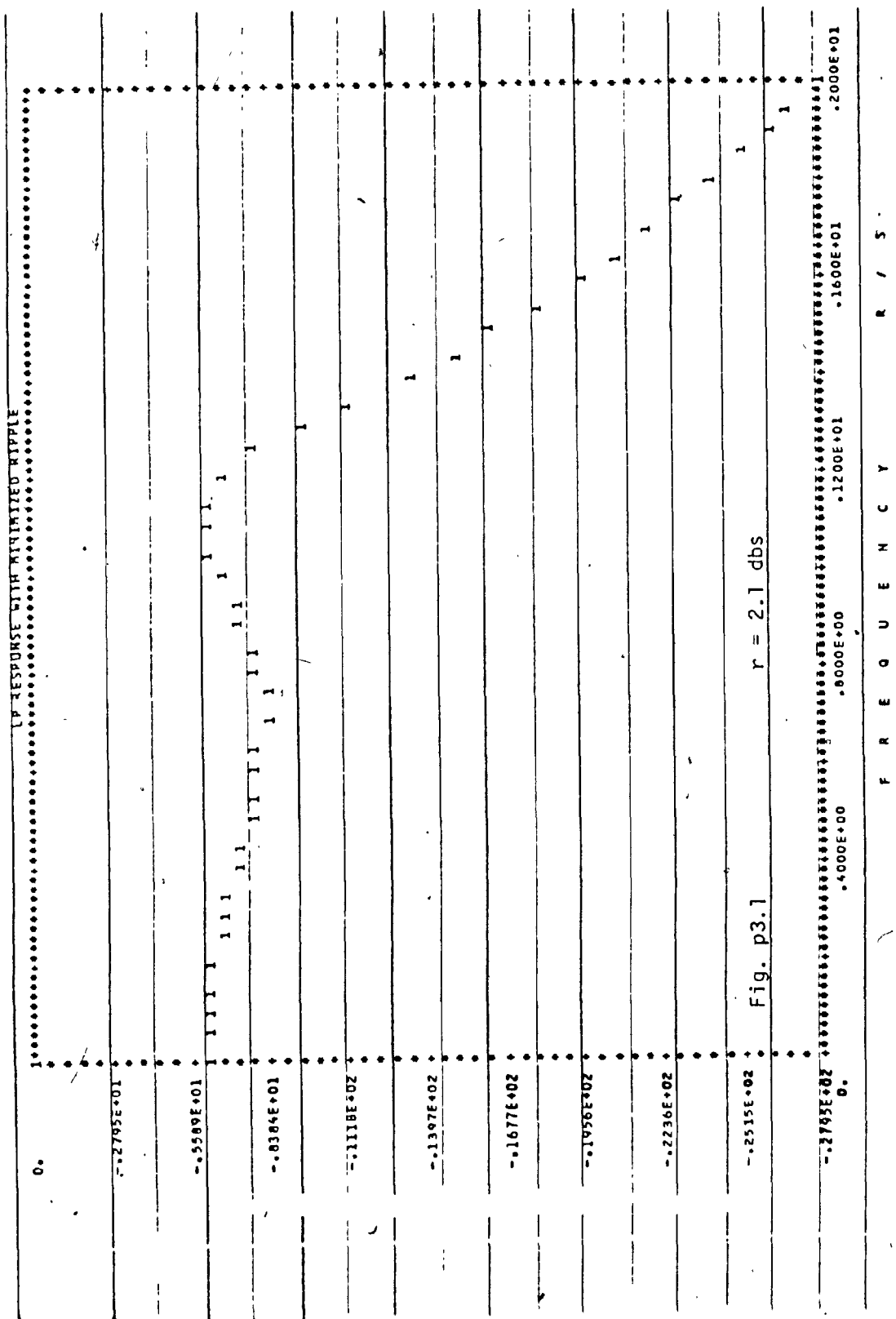
The element values given by this program are in 12 digits but for our use for plotting the response we have rounded it to 4 digits. This explains the little disparity between the value of the ripple given by this program and the ripple obtained by plotting the frequency response.

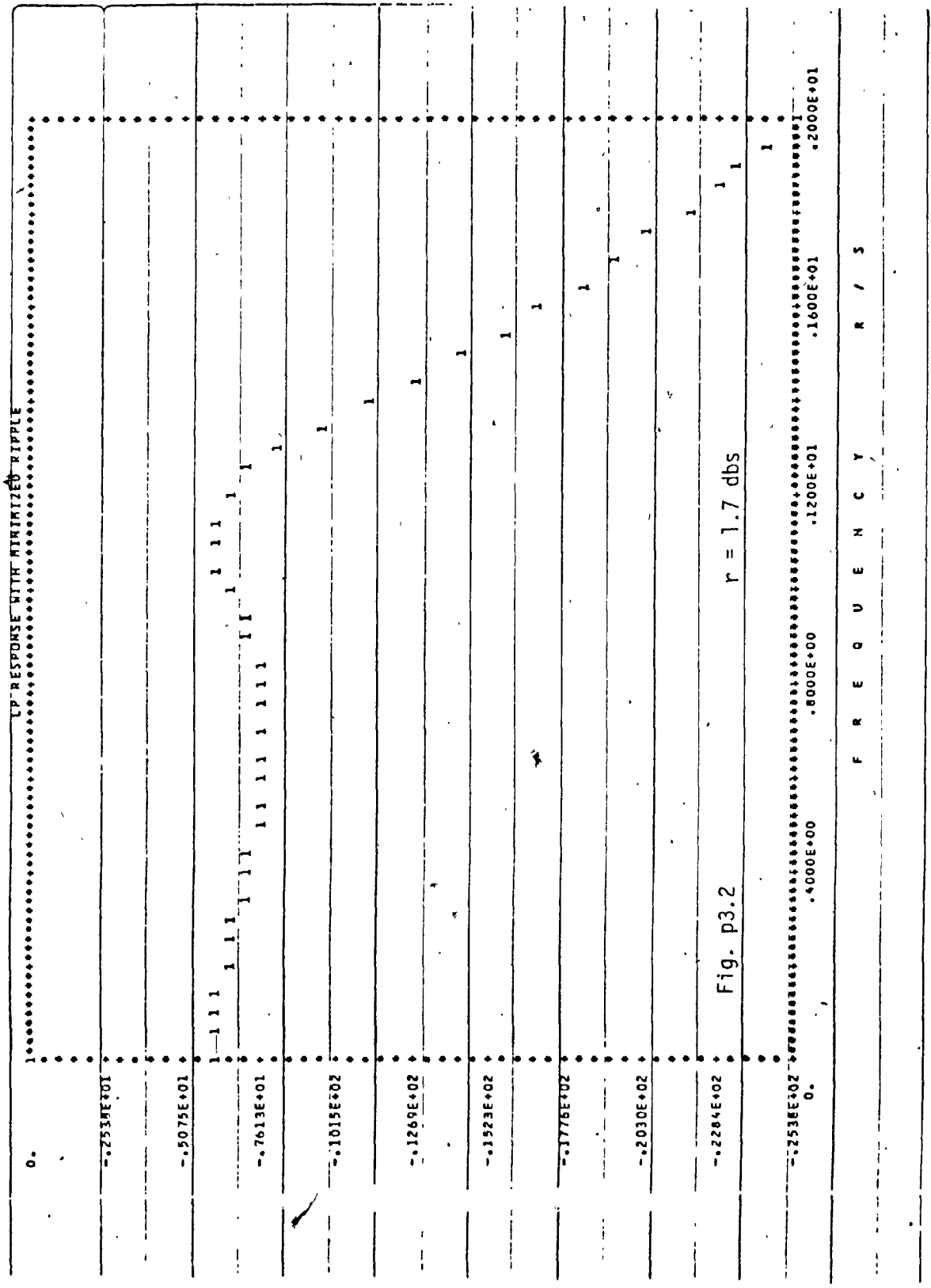
The program 'AFTAB' prints the parameter values and the corresponding ripple value after every iteration. The program in general minimizes the ripple in every iteration as can be seen in Fig. p3. We can choose the set of element values which has ripple close to our requirement but we have to neglect the set of values with negative element values.

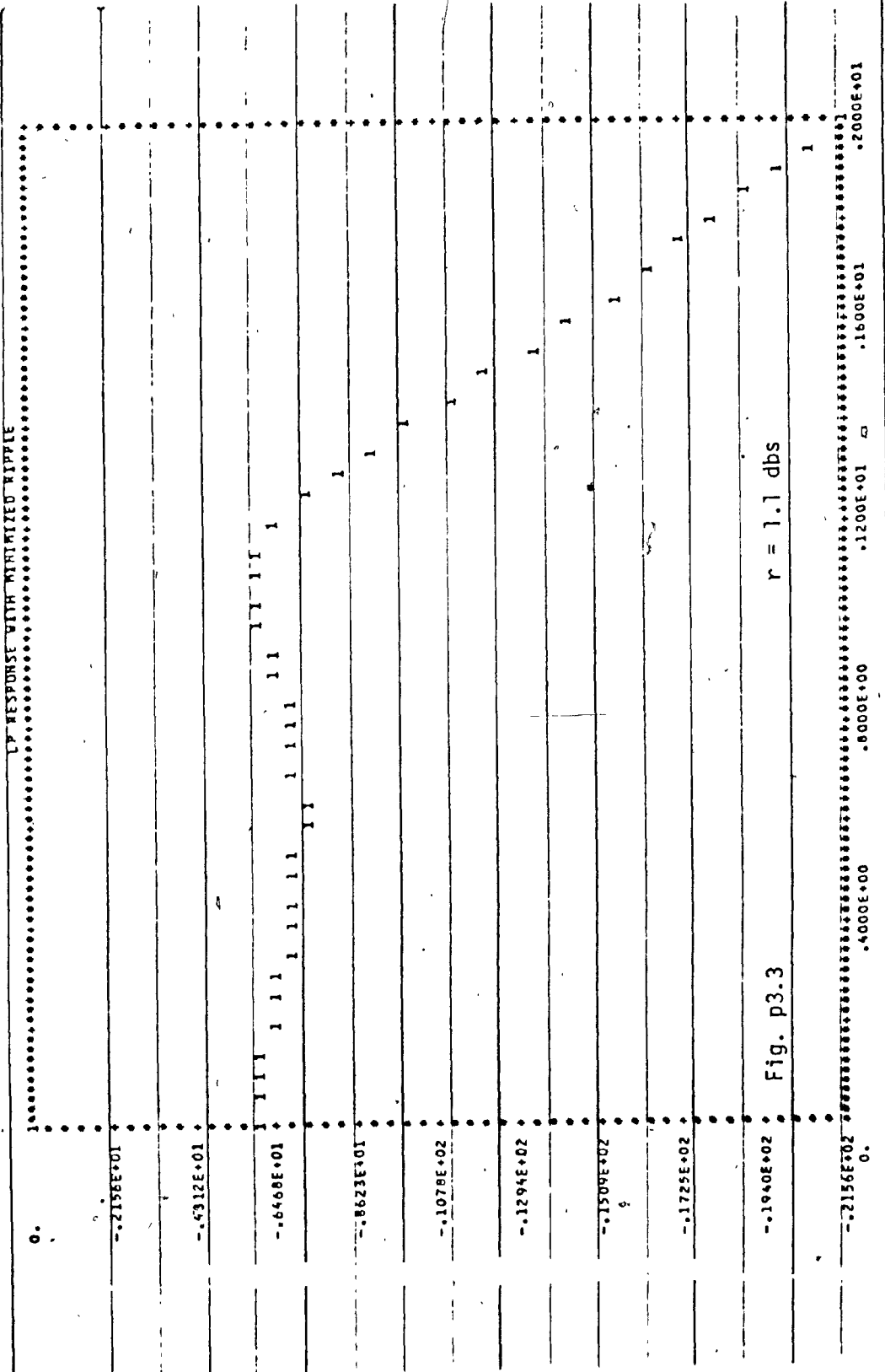
After selecting a set of values we plot the frequency response with these element values. If the ripple is more than our requirement then we can try the next set of element values and if it is less than our requirement we can try the previous set of values.

As can be seen from the computer plots of Fig. p3.1 to Fig.p3.4 we were able to obtain the low-pass response with ripples of 2.1 db in Fig.p3.1, 1.7 db in Fig.p3.2, 1.1 db in Fig.p3.3 and almost a flat response in Fig.p3.4.

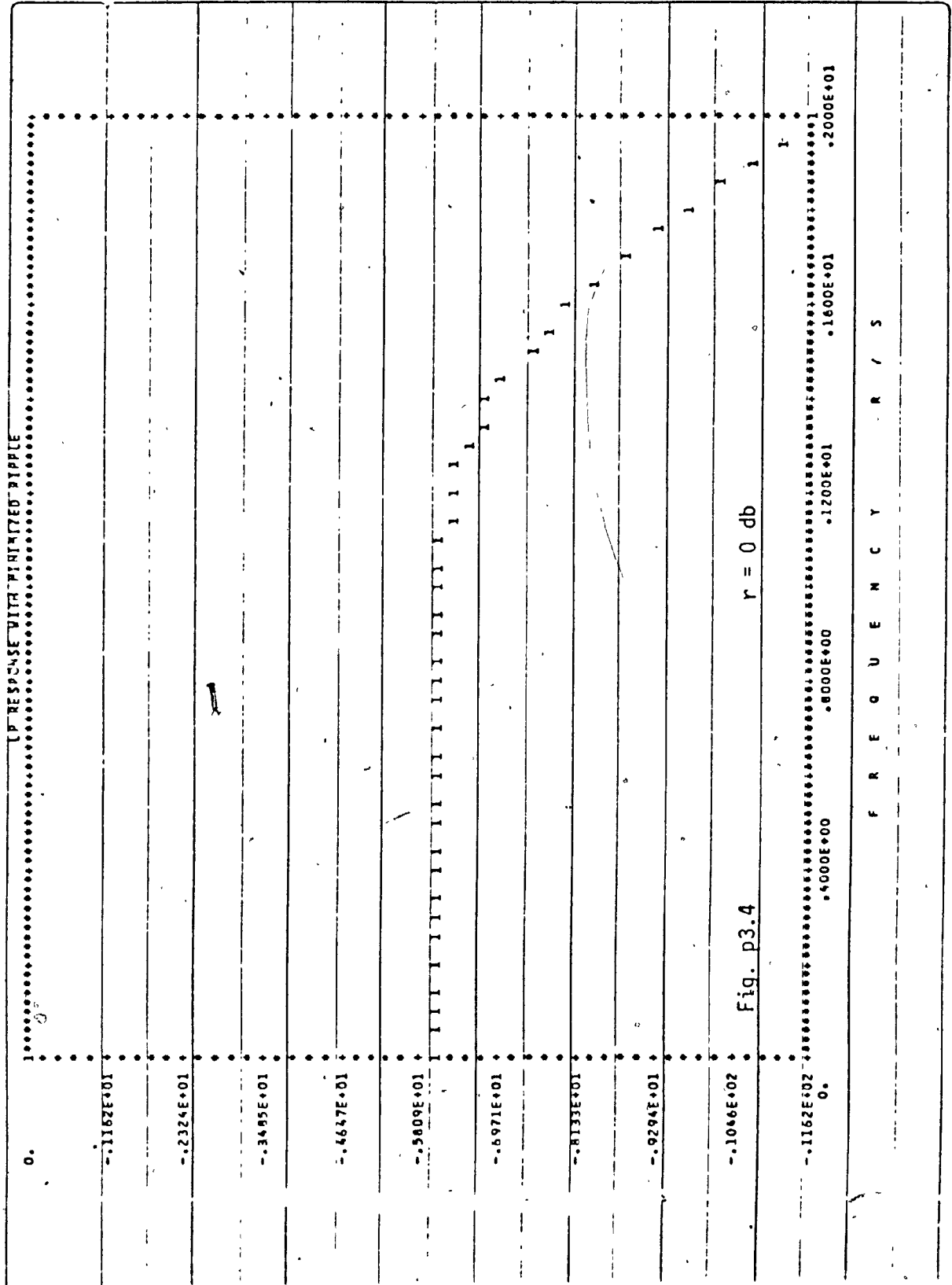
The ripple values obtained from the response are almost equal to the corresponding ripple given by the program 'AFTAB'.







F R E Q U E N C Y R / S



### 2.3.1 MIXED LUMPED-DISTRIBUTED STRUCTURES WITH TWO LUMPED ELEMENTS

In the preceeding sections we have discussed a Mixed Lumped-Distributed structure with four lumped elements. In this section we will discuss a Mixed Lumped-Distributed structure with two lumped elements, a series inductor and a shunt capacitor as shown in Fig. 2.3.1.

The overall chain parameters of this structure is derived in section 1.2.2 and are,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(sd) + \frac{Z_2}{Z} \sinh(sd) + y_1 z \sinh(sd) & z \sinh(sd) \\ + y_1 z_2 \cosh(sd) & + z_2 \cosh(sd) \\ \frac{1}{Z} \sinh(sd) + y_1 \cosh(sd) & \cosh(sd) \end{bmatrix}$$

For reasons described in section 2.2.1 we had to resort to computer-aided plot of the low-pass frequency response of the structure of Fig. 2.3 to investigate its behavior.

We obtained two frequency responses, one corresponding to the Butterworth element values and the second corresponding to Chebyshev element values.

#### 2.3.1.1 LP FILTERS WITH BUTTERWORTH ELEMENT VALUES

The low-pass response of the structure of Fig. 2.3.1 was obtained by substituting Butterworth element values for  $n = 2$  and  $r = 1$ , for lumped elements.

PROGRAM LPN2

73/174 OPT=1

FTN 4.8+498

80/03/05.

```

1      PROGRAM LPN2(INPUT,OUTPUT)
      C      THIS PROGRAM GIVES THE LP FREQUENCY PLOT OF
      C      A TRANSMISSION LINE WITH SERIES INDUCTOR
5      C      AT ONE END AND SHUNT CAPACITOR AT THE
      C      OTHER END, DOUBLY TERMINATED IN 1 OHM
      C      RESISTANCE.

      COMPLEX S,HS,ZS,ZC,HS1,Y1,Z2
10     DIMENSION X(201),Y(201,1),A(160),IMAG4(5151)
      REAL L2
      READ*,C1,L2,D,Z
      PRINT 11
11     FORMAT(1H1,3(/))
15     PRINT*, "      ELEMENT VALUES :
      PRINT*, "      C1= ",C1," L2= ",L2," D= ",D," Z= ",Z
      PRINT*, "-----"
20     PRINT 27
      PRINT 22
22     FORMAT(15X,"*", " FREQUENCY",3X,"*",4X,"ATTENUATION (DB)", " *")
      PRINT 27
27     FORMAT(15X,40(1H*))
25     DO 10 I=1,201,5
      W=(I-1)*0.01
      S=CMPLX(0.0,W)
      ZS=CMPLX(0.0,SIN(W*D))
      ZC=CMPLX(COS(W*D),0.0)
30     Y1=C1*S
      Z2=L2*S
      HS=Z2*Y1*ZC+((Z2/Z+Y1*Z)*ZS+(Z2+Y1)*ZC)+2*ZC+(Z+1/Z)*ZS
      HS1=1/HS
      F1=CABS(HS1)
35     Y(I,1)=20*ALOG10(F1)
      X(I)=W
      PRINT 33,W,Y(I,1)
33     FORMAT(15X,"*",2X,F4.2,8X,"*",7X,F12.8,4X,"*")
10     CONTINUE
      PRINT 27
40     READ 15,(A(I),I=1,160)
15     FORMAT(80A1)
      CALL USPLN(X,Y,201,1,1,201,A,IMAG4,IER)
45     STOP
      END

```



These responses are shown in Fig. p4.1 to Fig. p4.6 against increasing value of the time delay 'D' starting from zero and incremented by 0.2 in each step.

The Fig. p4.1, which is the low-pass frequency response of the structure of Fig. 2.3.1, with 'D' as zero, gives a 3 db frequency  $\omega_r$  of 1 r/s. The magnitude of attenuation is about 18 db.

In Fig. p4.2 with D as 0.2 the 3 db frequency  $\omega_r$  drops to 0.89 r/s and the magnitude of attenuation rises to 20 dbs.

For D = 0.4, the  $\omega_r$  is 0.83 r/s and the magnitude of attenuation is 20 dbs.

For D = 0.6, the  $\omega_r$  is 0.76 r/s and the magnitude of attenuation is 18.5 dbs.

For D = 0.8, the  $\omega_r$  is 0.73 r/s and the magnitude of attenuation is 16 dbs.

For D = 1.0, the  $\omega_r$  is 0.70 r/s and the magnitude of attenuation is 14 dbs.

### 2.3.1.2 LP FILTERS WITH CHEBYSHEV ELEMENT VALUES

The low-pass frequency response of the structure of Fig. 2.3.1 was obtained using Chebyshev element values for  $n = 3$ . For reasons discussed in section 2.2.1.2 we had to choose this set of Chebyshev element values. In our case we have two lumped elements in the structure of Fig. 2.3.1. We chose one value for the series inductor and one for the shunt capacitor.

The response of the structure with Chebyshev element

UP D. CURVE WITH MULTIPLY ELMENT VALUES FOR N=2 AND D=0.0

2

# THE

— 222 —

0-64478-9

— 22,180.1

12345

•

11

11-11-61

[illegible]

●  
●  
●  
●  
●  
●  
●  
●  
●

●●●●●●●●●●

● ● ● ● ● ● ● ● ●

●●●●●●●

Fig. p4.1

$$\omega = 1 \text{ r/s}$$

0 1 2 3 4 5 6 7 8 9

LP RESPONSE WITH BUTTERWORTH ELEMENT VALUES FOR  $N=2$  AND  $D=0.2$

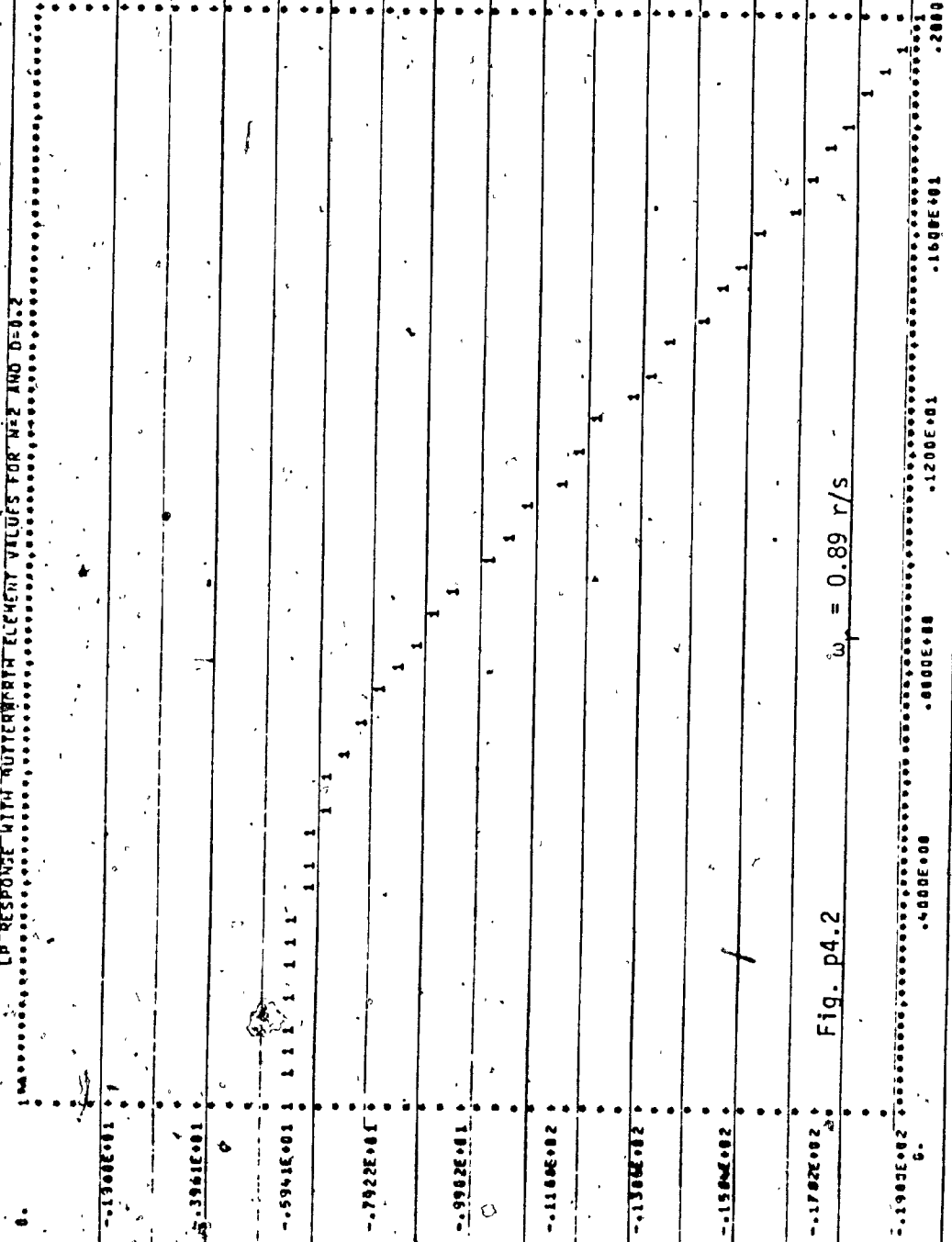


Fig. p4.2

$\omega_p = 0.89 \text{ r/s}$

F R E Q U E N C Y R / S

UP RESPONSE WITH BUTTERFLY ELEMENT VALUES FOR N=2 AND D=0.6

9.

**FD-398b1-**

-.3971E+01

-5957E001

1043640

1003E01

-11915-02

## EXERCISES

15851

1  
2  
3  
4  
5  
6  
7

$$\omega_s = 0.83 \text{ r/s}$$

Fig. p4.3

20039067-

10-30002-10-30091

1200E-01

700-308917

2009E-01

FR E D E R I C Y R / S



LP RESPONSE WITH DIFFERENTIAL ELEMENT VALUES FOR H2 AND D2-J.8

0. ....

-.1591E+01

-.3183E+01

-.4774E+01

-.6366E+01

-.7957E+01

-.9548E+01

-.1114E+02

-.1273E+02

-.1432E+02

-.1591E+02

Fig. p4.5

$\omega_r = 0.73 \text{ r/s}$

.2000E+01

.1600E+01

.1200E+01

.0800E+00

.0400E+00

F R E Q U E N C Y R / S

LP RESPONSE WITH BUTTERWORTH ELEMENT VALUES FOR $N=2$ AND $D=1$									
0.									
-.1397E+01									
-.2794E+01									
-.4192E+01									
-.5589E+01									
-.6986E+01									
-.8383E+01									
-.9780E+01									
-.1118E+02									
-.1257E+02									
-.1397E+02									
0.									
-.4000E+00									
.8000E+00									
.1200E+01									
.1600E+01									
.2000E+01									

Fig. p4.6  $\omega_p = 0.70$  r/s

F R E Q U E N C Y R / S

[illegible]



LP RESPONSE WITH CHERYSSEV ELEMENT VALUES FOR N=2 AND D=0.2	
0.	1
-.2012E+01	
-.4025E+01	
-.6037E+01	1 1 1 1 1 1
-.8049E+01	1 1 1 1 1 1
-.1006E+02	1 1 1 1 1 1
-.1207E+02	1 1 1 1 1 1
-.1409E+02	1 1 1 1 1 1
-.1610E+02	1 1 1 1 1 1
-.1811E+02	1 1 1 1 1 1
-.2012E+02	1 1 1 1 1 1
0.	1 1 1 1 1 1
-.4000E+00	
-.1208E+01	
-.1600E+01	
-.2000E+01	
F R E Q U E N C Y R / S	

Fig. p5.2





CLP RESPONSE WITH CHEBYSHEV ELEMENT VALUES FOR  $N=2$  AND  $D=0.8$

U

- . 1635E + 01

10-30236-01

100390640-

10431590-

-01763-01

10-9011E-01

20035711--

- 1304502 -

-28472E+02

Fig. p5.5

-	.1635E+02	.
B.	.400CE+08	
	.800DE+00	
	.128OE+01	
	.150OE+01	
	.200OE+01	

F R E Q U E N C Y / S



values is shown in Fig. p5.1 to Fig. p5.6 which are plotted against increasing value of D starting from zero.

As we can see the responses are flat that is they do not have any passband ripple for any value of D.

Thus it is not possible to design a Chebyshev filter for this case.

### 2.3.2 BUTTERWORTH FILTER DESIGN

The design procedure described in section 2.2.2.1 is the procedure required to design a Butterworth filter in this case. In Butterworth filters the cutoff frequency has to be 1 r/s irrespective of the number of elements in the network. So we can choose the time delay D of our requirement and normalize the cutoff frequency to 1 r/s for that response.

### 2.4.1 THE MIXED LUMPED-DISTRIBUTED LOW-PASS STRUCTURES WITH SIX LUMPED ELEMENTS

The structure is shown in Fig. 2.4.1. The chain parameters of this structure is derived in section 1.2.3 and from equation 1.2.3 they are

$$\begin{aligned}
 A = & (1 + z_6 y_5) \cosh(sd) + ((1 + z_6 y_5) z_4 + z_6) \frac{1}{Z} \sinh(sd) + ((1 + z_6 y_5) z \sinh(sd) \dots \\
 & \dots + ((1 + z_6 y_5) z_4 + z_6) \cosh(sd)) y_3 + (((1 + z_6 y_5) z_4 + z_6) \frac{1}{Z} \sinh(sd) \dots \\
 & \dots + ((1 + z_6 y_5) z \sinh(sd) + ((1 + z_6 y_5) z_4 + z_6) \cosh(sd)) y_3) z_2 \dots \\
 & \dots + (1 + z_6 y_5) z \sinh(sd) + ((1 + z_6 y_5) z_4 + z_6) \cosh(sd)) y_1
 \end{aligned}$$

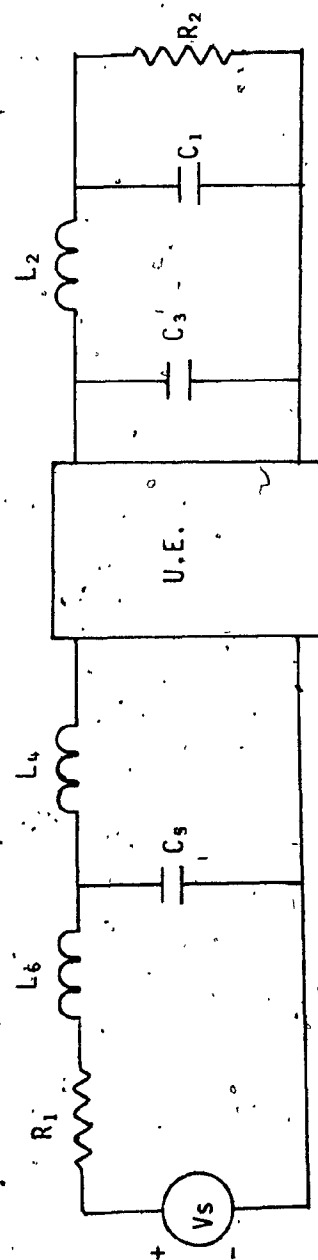


Fig. 2.4.1

$$\begin{aligned}
 B = & ((1 + z_6 y_5) \cosh(sd) + ((1 + z_6 y_5) z_4 + z_6) \frac{1}{z} \sinh(sd) \dots \\
 & \dots + ((1 + z_6 y_5) z \sinh(sd) + ((1 + z_6 y_5) z_4 + z_6) \cosh(sd)) y_3 z_2 \dots \\
 & \dots + (1 + z_6 y_5) z \sinh(sd) + ((1 + z_6 y_5) z_4 + z_6) \cosh(sd)
 \end{aligned}$$

$$\begin{aligned}
 C = & y_5 \cosh(sd) + (1 + y_5 z_4) \frac{1}{z} \sinh(sd) + (y_5 z \sinh(sd) \dots \\
 & \dots + (1 + y_5 z_4) \cosh(sd)) y_3 + ((y_5 \cosh(sd) + (1 + y_5 z_4) \frac{1}{z} \sinh(sd) \dots \\
 & \dots + (y_5 z \sinh(sd) + (1 + y_5 z_4) \cosh(sd)) y_3) z_2 + y_5 z \sinh(sd) \dots \\
 & \dots + (1 + y_5 z_4) \cosh(sd)) y_1
 \end{aligned}$$

$$\begin{aligned}
 D = & (y_5 \cosh(sd) + (1 + y_5 z_4) \frac{1}{z} \sinh(sd) + (y_5 z \sinh(sd) \dots \\
 & \dots + (1 + y_5 z_4) \cosh(sd)) y_3) z_2 + y_5 z \sinh(sd) + (1 + y_5 z_4) \cosh(sd)
 \end{aligned}$$

Here also we will proceed in the manner described in section 2.2.1 and section 2.3.1. We will plot the low-pass frequency response for the structure of Fig. 2.4.1 with Butterworth and Chebyshev element values. Then observing the response we will formulate the design procedure for Butterworth and Chebyshev filters.

#### 2.4.1.1 LP FILTERS WITH BUTTERWORTH ELEMENT VALUES

Butterworth element values corresponding to  $n = 6$  and  $r = 1$  were substituted for lumped elements of structure of Fig. 2.4.1 to obtain the low-pass response.

These responses are plotted against increasing value of time delay 'D' and are shown in Fig. p6.1 to Fig. p6.7.

For  $D = 0$ , the 3 db frequency  $\omega_r$  is 1 r/s and the magnitude of attenuation is about 42 db as seen in Fig. p5.1.

For  $D = 0.2$ , the  $\omega_r$  is 0.93 r/s and the magnitude of attenuation



PROGRAM LPN6 73/174 OPT=1

FTN 4.8+498

80/03/05.

```

1      PROGRAM LPN6(INPUT,OUTPUT)
C      THIS PROGRAM GIVES THE LP FREQUENCY RESPONSE OF
C      MIXED LUMPED-DISTRIBUTED STRUCTURE WITH SIX LUMPED
C      ELEMENTS. THE CASCADED NETWORK IS DOUBLY TERMINATED
C      IN 1 OHM RESISTOR.
C      THE TRANSFER FUNCTION OF THE NETWORK IS FOUND TO
C      BE,  $TRF = 1/(A+B+C+D)$ . THEREFORE HS1, HS2, HS3, HS4
C      REPRESENT THE CHAIN PARAMETERS A, B, C, D OF THE
C      NETWORK IN THAT ORDER.

      COMPLEX S=HS1, HS2, HS3, HS4, HS5, Z8, ZC, Y1, Z2, Y3, Z4, Y5, Z6
      DIMENSION X(201), Y(201,1), A(140), IMAG4(5151)
      REAL L2, L4, L6, Z, D
15      READ*, C1, L2, C3, L4, C5, L6, D, Z
      PRINT 11
11      FORMAT(1H1,5(/))
      PRINT*, "ELEMENT VALUES"
      PRINT*, "-----"
20      PRINT*, "C1= ", C1, " L2= ", L2, " C3= ", C3, " L4= ", L4
      PRINT*, "C5= ", C5, " L6= ", L6, " D= ", D, " Z= ", Z
      PRINT 17
17      FORMAT(3X,35(1H-),//)
      PRINT 27
27      FORMAT(15X,38(1H-))
      DO 10 I=1,201,5
30      W=(I-1)*0.01
      S=CMPLX(0.0,W)
      Z8=CMPLX(0.0,SIN(W*D))
      ZC=CMPLX(COS(W*D),0.0)
      Y1=C1*S
      Z2=L2*S
      Y3=C3*S
      Z4=L4*S
      Y5=C5*S
      Z6=L6*S

      HS1=((1+Z4*Y5)*ZC+((1+Z6*Y5)*Z4+Z6)*Z5/Z+
      0*((1+Z6*Y5)*Z8*Z5+((1+Z6*Y5)*Z4+Z6)*ZC)*Y3
      0*((1+Z6*Y5)*ZC+((1+Z6*Y5)*Z4+Z6)*Z5/Z+
      0*((1+Z6*Y5)*Z8*Z5+((1+Z6*Y5)*Z4+Z6)*ZC)*Y3
      0)*Z2+((1+Z6*Y5)*Z8*Z5+((1+Z6*Y5)*Z4+Z6)*ZC)*Y1

      HS2=((1+Z6*Y5)*ZC+((1+Z6*Y5)*Z4+Z6)*Z5/Z+
      0*((1+Z6*Y5)*Z8*Z5+((1+Z6*Y5)*Z4+Z6)*ZC)*Y3
      0)*Z2+((1+Z6*Y5)*Z8*Z5+((1+Z6*Y5)*Z4+Z6)*ZC)

      HS3=Y5*ZC+((1+Y5*Z4)*Z8/Z+(Y5*Z8*Z5+((1+Y5*Z4)*
      0)*ZC)*Y3+((Y5*ZC+((1+Y5*Z4)*Z5/Z+(Y5*Z8*Z5+
      0*((1+Y5*Z4)*ZC)*Y3)*Z2+Y5*Z8*Z5+((1+Y5*Z4)*ZC)*Y1

      HS4=(Y5*ZC+((1+Y5*Z4)*Z8/Z+(Y5*Z8*Z5+((1+Y5*Z4)*
      0)*ZC)*Y3)*Z2+Y5*Z8*Z5+((1+Y5*Z4)*ZC)

```

PROGRAM LPN6 73/174 OPT=L

FTN 4.8+498

80/03/05.

```

60      HS5=1/(HS1+HS2+HS3+HS4)
      F1=CABS(HS5)
      Y(I,1)=20*ALOG10(F1)
      X(I)=W
33      PRINT 33,W,Y(I,1)
      FORMAT(15X,"S",2X,F4.2,8X,"S",7X,F12.8,2X,"S")
10      CONTINUE
      PRINT 27
      READ 15,(A(I),I=1,140)
15      FORMAT(80A1)
      CALL USPLN(X,Y,201,1,1,201,A,IMAG4,IER)
      STOP
      END
70

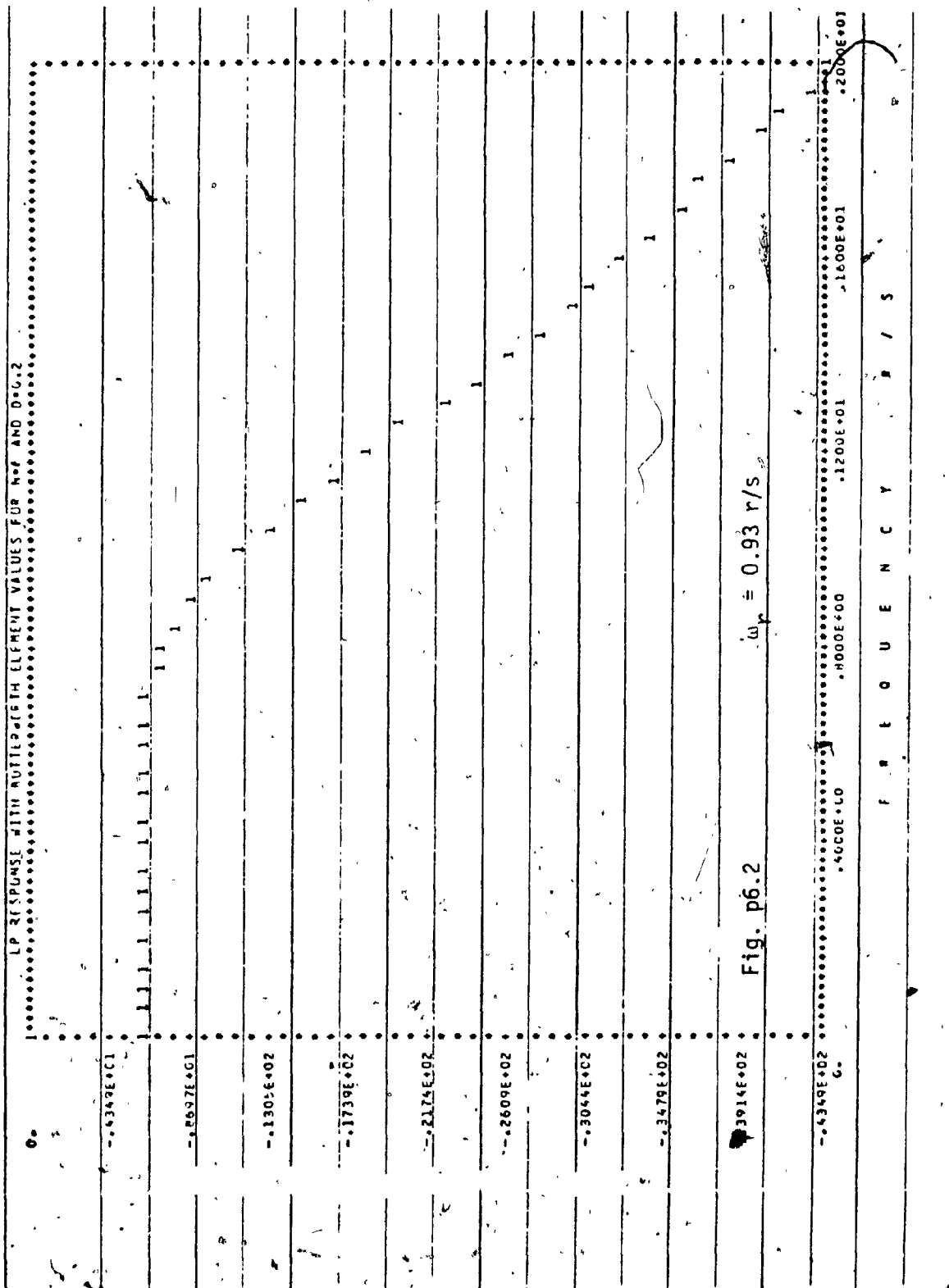
```

LP RESPONSE WITH FILTERED WORTH ELEMENT VALUES FOR N=6 AND D=0.0									
0.	1	1	1	1	1	1	1	1	1
-.4214E+01	1	1	1	1	1	1	1	1	1
-.8429E+01	1	1	1	1	1	1	1	1	1
-.1264E+02	1	1	1	1	1	1	1	1	1
-.1684E+02	1	1	1	1	1	1	1	1	1
-.2107E+02	1	1	1	1	1	1	1	1	1
-.2529E+02	1	1	1	1	1	1	1	1	1
-.2950E+02	1	1	1	1	1	1	1	1	1
-.3372E+02	1	1	1	1	1	1	1	1	1
-.3793E+02	1	1	1	1	1	1	1	1	1
-.4214E+02	1	1	1	1	1	1	1	1	1
0.	1	1	1	1	1	1	1	1	1
.4CCOE+00	1	1	1	1	1	1	1	1	1
.9030E+00	1	1	1	1	1	1	1	1	1
.1200E+01	1	1	1	1	1	1	1	1	1
.1600E+01	1	1	1	1	1	1	1	1	1
.2000E+01	1	1	1	1	1	1	1	1	1
F I L T E R E D W O R T H E L E M E N T V A L U E S									

Fig. p6.1

 $\omega_p = 1.0 \text{ r/s}$

LP RESPONSE WITH MULTIPLE ELEMENT VALUES FOR HOF AND D06.2





LV RESPONSE WITH NUTTE-ALPHA ELEMENT VALUES FOR N=6 AND D=0.6									
1	1	1	1	1	1	1	1	1	1
-0.4171E+01	1	1	1	1	1	1	1	1	1
-0.3343E+01	1	1	1	1	1	1	1	1	1
-0.1251E+02	1	1	1	1	1	1	1	1	1
-0.1669E+02	1	1	1	1	1	1	1	1	1
-0.2086E+02	1	1	1	1	1	1	1	1	1
-0.2503E+02	1	1	1	1	1	1	1	1	1
-0.2920E+02	1	1	1	1	1	1	1	1	1
-0.3337E+02	1	1	1	1	1	1	1	1	1
-0.3754E+02	1	1	1	1	1	1	1	1	1
-0.4171E+02	1	1	1	1	1	1	1	1	1
0.	1	1	1	1	1	1	1	1	1
0.4000E+00	1	1	1	1	1	1	1	1	1
0.8000E+00	1	1	1	1	1	1	1	1	1
0.1200E+01	1	1	1	1	1	1	1	1	1
0.1600E+01	1	1	1	1	1	1	1	1	1
0.2000E+01	1	1	1	1	1	1	1	1	1
F R E Q U E N C Y R S									

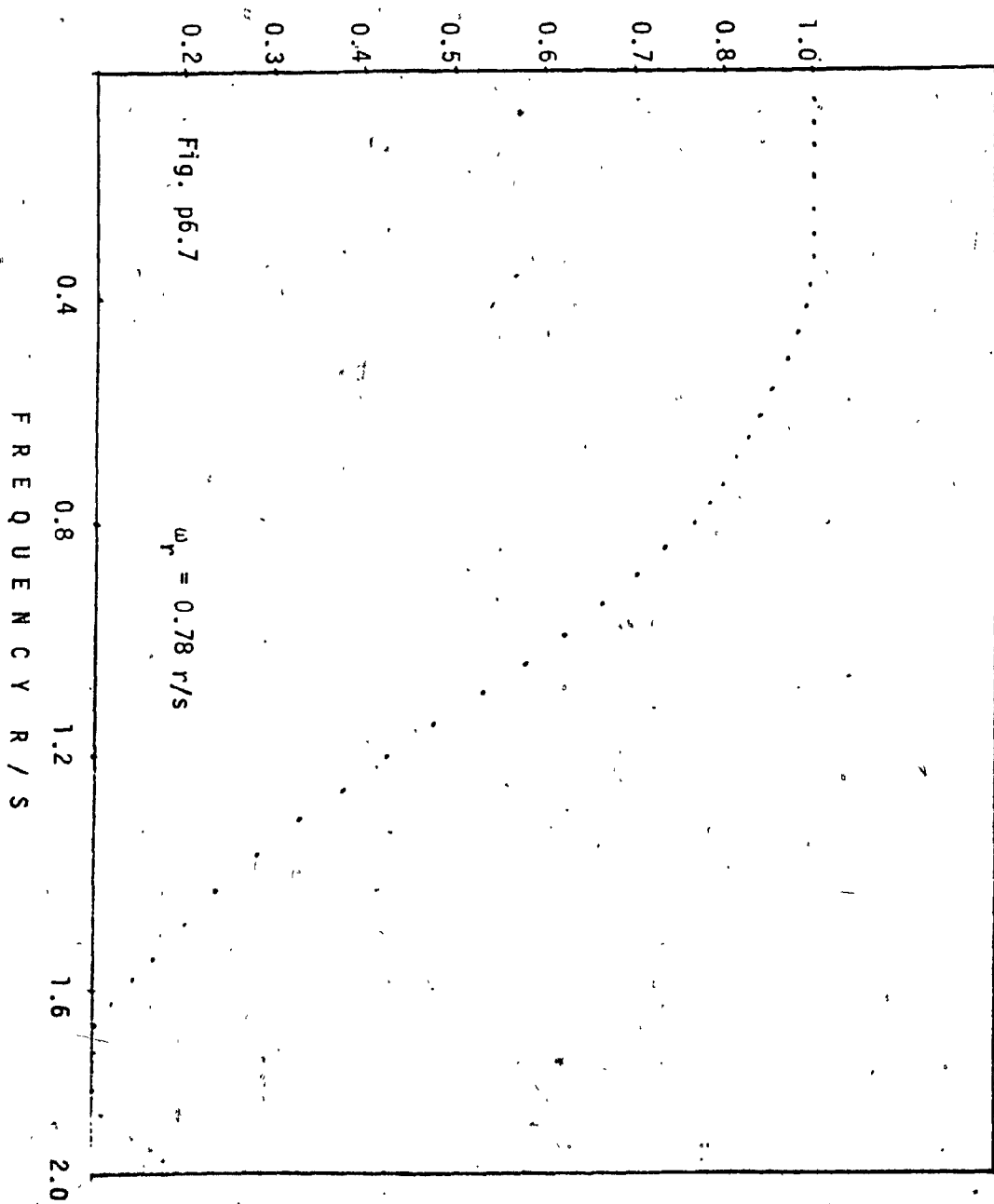
Fig. p6.4

 $\omega_p = 0.82 \text{ r/s}$

[illegible]



LP Response with Butterworth Element Values for  $N = 6$  and  $D = 1.1$















.....

-5329E+02

 $r = 0.75 \text{ db}$ 

2

$\frac{1}{T} \ln \frac{1}{1 - \frac{1}{T}}$

3 0 3 1 1 3 7

attenuation is 64 dbs.

For  $D = 0.6$ ,  $\omega_r$  is 0.83 r/s,  $r$  is 0.9 db and the magnitude of attenuation is 63 dbs.

For  $D = 0.8$ ,  $\omega_r$  is 0.74 r/s,  $r$  is 0.8 db and the magnitude of attenuation is 60 dbs.

For  $D = 1.0$ ,  $\omega_r$  is 0.69 r/s,  $r$  is 0.75 db and the magnitude of attenuation is 52 dbs.

#### 2.4.2.1 BUTTERWORTH FILTER DESIGN

In sections 2.2.1.2 and 2.3.1.2 we have discussed the Butterworth filter design procedure with four and two lumped elements respectively. Similar procedure will be carried out here with six lumped elements. We choose a value for the time delay according to the requirement and normalize the corresponding cutoff frequency to 1 r/s.

#### 2.4.2.2. CHEBYSHEV FILTER DESIGN

The passband ripple was above 2 dbs when there were four lumped elements (section 2.2.1.2). That fact necessitated the development of ripple minimizing program. In the present case the passband ripple varies between 0.75 db to 1.2 dbs. Therefore no design procedures are necessary and we can choose the required ripple and the corresponding 'D' value will be obtained. Our choice of ripple in this case is limited and we can have the ripple values between 0.75 db and 1.2 dbs.

## 2.5 RESULTS AND DISCUSSION

### 2.5.1 RESPONSE WITH BUTTERWORTH ELEMENT VALUES

The low-pass response with Butterworth element values for  $n = 4$  as shown in the figures of p1.1 to p1.6 gives a value of  $\omega_p$  of 1 r/s when the group delay is zero. The magnitude of attenuation at this point is about 30 dbs. As we increase the value of 'D', the time delay the magnitude of attenuation drops to about 16 dbs for  $D = 1$ .

The value of  $\omega_p$  also drops and is 0.74 r/s at  $D = 1$ .

The response with Butterworth element values for  $n = 2$  as shown in figures of p4.1 to p4.6 again gives a value of  $\omega_p$  of 1 r/s for  $D = 0$ . As in the earlier case the value of  $\omega_p$  drops with the increase in  $D$  and reaches 0.70 r/s for  $D = 1$ . The magnitude of attenuation however does not vary much. It remains almost constant for values of 'D' from 0 to 0.6. For values of 'D' higher than 0.6, the magnitude of attenuation drops to 13 dbs for value of  $D = 1$ . As we increase the value of 'D' from 0.6 the magnitude of attenuation starts dropping at the frequency of 1.75 r/s for  $D = 0.8$  and at 1.45 r/s for  $D = 1$ .

The value of  $\omega_p$  as shown is again 1 r/s for  $n = 6$  when  $D = 0$ . As shown in figures p5.1 to p5.6 for  $n = 6$ , the magnitude of attenuation remains almost constant at 40 dbs till the value of 'D' is increased to 0.6. There is a sudden drop in the magnitude of attenuation when 'D' is increased from 0.6 to 0.8. The attenuation at  $D = 0.8$  is 20 dbs. The magnitude of attenuation at  $D = 1$  rises to 30 dbs.



### 2.5.2 RESPONSE WITH CHEBYSHEV ELEMENT VALUES

The response with Chebyshev element values for  $n = 4$  is obtained for values of  $D$  higher than 0.2. At  $D = 0.4$  the value of  $\omega_p$  is 1.56 r/s and the passband ripple is 5.5 dbs.

As shown in figures p2.1 to p2.6, the cutoff frequency drops to almost 1 r/s as the value of  $D$  is increased to 1. The passband ripple decreases with the increase in  $D$  and reaches 2 dbs for  $D = 1$ . The response obtained in this case is not equi-ripple.

We were not able to obtain Chebyshev type response for  $n = 2$  as shown in Figures p5.1 to p5.6. It is due to the fact that we have only two lumped element in this case as shown in Fig. 2.3.1. The Chebyshev filter table does not give element values for even orders for  $r = 1$ .

However we were able to obtain Chebyshev type response for  $n = 6$  as can be seen in figures p7.1 to 7.6. There is not much change in the ripple. When we increase the value of  $D$  from 0 to 1, the ripple reduces gradually from 1.2 dbs to 0.75 db. The cutoff frequency also drops from 0.97 r/s to 0.74 r/s.

### 2.5.3 RESPONSES WITH MINIMIZE RIPPLE

The results shown in the Fig. of p3 gives the various set of parameter values. As discussed before the actual frequency response from these parameters slightly differ in the sense that the passband ripple is not exactly the same for the reasons given in section 2.2.3. In some cases the difference was negligible. From the plots of figure p3.1 to

p3.4 we can see that the ripple in Fig. p3.1 is about 2.1 dbs where the minimum ripple in the original plot was 4.2 dbs. In figures p3.2 and in p3.3 the ripple is less than 2 db. and it is 1 db in Fig. p3.3. The figure of p3.4 shows that we were even able to obtain a maximally flat response with the use of parameters given by the ripple minimization program.

This ripple minimization program can be improved to control the cutoff frequency. That is to keep the cutoff frequency within certain level. From these results it can be seen that lower the cutoff frequency the higher is the magnitude of attenuation.

## CHAPTER 3

### HIGH-PASS NETWORKS

#### 3.1 GENERAL HIGH-PASS NETWORKS

High-pass filters are designed by inverting the response requirements, so that they become requirements on a low-pass filter. Low-pass filter meeting this new requirement can be readily transformed into a high-pass filter which meets the original requirement.

Alternatively, we can consider that our catalog of low-pass response curves can be read as high-pass response curves merely by reading the attenuation as recorded and taking the reciprocal of the frequencies.

We here used the frequency transformation of replacing  $\omega$  by  $1/\omega$ . Then, whatever attenuation was given by the low-pass filter at  $\omega$ , is now given by the high-pass filter at  $1/\omega$ .

For the plot of high-pass response the program for the frequency plot in low-pass can be used with above substitution.

Since we are not dealing with purely lumped structures we may not be able to obtain high-pass response by merely inverting the response requirements due to the presence of Transmission Line.

The low-pass structures of Fig. 2.2.1, Fig. 2.3.1 and Fig. 2.4.1 can be converted into high-pass structures by replacing the series inductances by capacitances and shunt capacitances by inductances.

Design procedures similar to those in low-pass can be followed to obtain the high-pass Butterworth and Chebyshev filters.

### 3.2 RESULTS AND DISCUSSION

#### 3.2.1 HIGH-PASS RESPONSE WITH BUTTERWORTH ELEMENT VALUES

The high-pass Butterworth response for  $n = 2$  gives a value of  $\omega_p$  of 1 r/s for  $D = 0$ , as in the case of low-pass. With the increase in 'D' the value of  $\omega_p$  increases. It reaches 1.4 r/s for the value of  $D = 1$ . The magnitude of attenuation as shown in figures p8.1 to 8.6 fluctuates in the neighborhood of 80 dbs.

The value of  $\omega_p$  is again 1 r/s for  $n = 4$  with  $D = 0$ . The value of  $\omega_p$  increases with the increase in 'D' and reaches 1.35 r/s for  $D = 1$ . The response with 'D' greater than 0.4 gives kinks (overshoot) near the 3 db frequency region. The magnitude of attenuation remains little above 105 dbs with some variations as shown in figures p9.1 to p9.6.

For  $n = 6$  the value of  $\omega_p$  is close to 1 r/s for  $D = 0$ . It is 0.98 r/s. For 'D' greater than 0.2 we again have kinks (overshoot) as in the previous case. The magnitude of attenuation is above 230 dbs in these cases as can be seen in figures p10.1 to p10.6.

#### 3.2.2 HIGH-PASS RESPONSE WITH CHEBYSHEV ELEMENT VALUES

The high-pass response with Chebyshev element values does not give passband ripple for  $n = 2$  as shown in figures p11.1 to p11.3. It gives Butterworth type of response and the value of  $\omega_p$  increases from 1.05 r/s when  $D = 0$  to 1.51 r/s for  $D = 1$ . The magnitude of attenuation remains around 85 dbs.

The value of  $\omega_p$  for  $n = 4$  is higher and it is 1.6 r/s.

PROGRAM HPN2

73/174 OPT=1

FTN 4.8+498

80/03/05.

```

1      PROGRAM HPN2(INPUT,OUTPUT)
C -    THIS PROGRAM GIVES THE HP FREQUENCY PLOT OF
C      A TRANSMISSION LINE WITH SERIES INDUCTOR
5      AT ONE END AND SHUNT CAPACITOR AT THE
C      OTHER END, DOUBLY TERMINATED IN 1 OHM
C      RESISTANCE.

      COMPLEX S,HS,ZS,ZC,HS1,Y1,Z2
      DIMENSION X(201),Y(201),A(160),IMAG4(5151)
      REAL L2
      READ*,C1,L2,D,Z
      PRINT 11
11     FORMAT(1H1,3(/))
15     PRINT*, " ELEMENT VALUES "
      PRINT*, "-----"
      PRINT*, " C1= ",C1," L2= ",L2," D= ",D," Z= ",Z
      PRINT*, "-----"
      PRINT*, "-----"
20     PRINT 27
      PRINT 22
22     FORMAT(15X,"*", " FREQUENCY",3X,"*",4X," ATTENUATION (DB)", " *")
      PRINT 27
27     FORMAT(15X,40(1H*))
25     DO 10 I=1,201,5
      W1=(I)*0.01
      W=1/W1
      S=CMPLX(0.0,W)
      ZS=CMPLX(0.0,SIN(W*D))
30     ZC=CMPLX(COS(W*D),0.0)
      Y1=C1*S
      Z2=L2*S
      HS=Z2*Y1+ZC*((Z2/Z+Y1*Z)*ZS+(Z2*Y1)*ZC)+2*ZC*(Z+1/Z)*ZS
      HS1=1/HS
35     F1=ABS(HS1)
      Y(I,1)=20*ALOG10(F1)
      X(I)=W1
      PRINT 33,W1,Y(I,1)
40     PRINT 27
      READ 15,(A(I),I=1,160)
10     CONTINUE
      PRINT 27
15     READ 15,(A(I),I=1,160)
      FORMAT(80A1)
      CALL USPLH(X,Y,201,1,1,201,A,IMAG4,IER)
45     STOP
      END

```

PROGRAM HPN4

73/174 OPT=1

FTN 4.8+498

80/03/05.

```

1      PROGRAM HPN4(INPUT,OUTPUT)
      C      THIS PROGRAM PLOTS THE HIGH-PASS FREQUENCY RESPONSE
      C      OF MIXED LUMPED-DISTRIBUTED STRUCTURE WITH FOUR
      C      LUMPED ELEMENTS USING THE PLOTTING SUBROUTINE 'USPLH'.
      C      THE ELEMENT VALUES ARE TAKEN FROM THE AVAILABLE
      C      FILTER TABLES FOR NORMALIZED CHEBYSHEV FUNCTION
      C      WITH 1 DB, RIPPLE AND NORMALIZED BUTTERWORTH FUNCTIONS.

10     COMPLEX S,HS1,HS2,HS3,HS4,HS5,ZS,ZC,Y1,Z2,Y3,Z4
      DIMENSION X(201),Y(201,1),A(160),IMAG4(5151)
      REAL C1,L2,C3,L4
      READ*,C1,L2,C3,L4,D,Z
      PRINT 11
15     11      FORMAT(1H1,3(/))
      PRINT 12
      12      FORMAT(15X,'ELEMENT VALUES')
      PRINT 13
      13      FORMAT(15X,14(1H-))
20     PRINT 14,C1,L2
      PRINT 15,C3,L4
      PRINT 16,D,Z
      14      FORMAT(15X,'C1=8,F7.4,8L2=8,F7.4)
      15      FORMAT(15X,'C3=8,F7.4,8L4=8,F7.4)
25     16      FORMAT(15X,'D=8,F5.3,8 Z=8,F5.3)
      PRINT 18
      18      FORMAT(/,15X,25(1H-),/)
      PRINT 27
      PRINT 22
      22      FORMAT(15X,'*', ' FREQ(R/S)',3X,'*',6X,'ATTENUATION (DB)', ' *')
30     27      FORMAT(15X,39(1H*))
      PRINT 27
      DO 10 I=5,201,5
      W1=(I)*0.01
35     W=1/W1
      S=CHFLX(0.0,W)
      ZS=CHFLX(0.0,SIN(W*D))
      ZC=CHFLX(COS(W*D),0.0)
      Y1=C1*S
40     Z2=L2*S
      Y3=C3*S
      Z4=L4*S
      HS1=((1+Z4*Y3)*ZC+(Z4*ZS/Z))*(1+Z2*Y1)+
      *(((1+Z4*Y3)*Z*ZS+Z4*ZC)*Y1))
45     HS2=((1+Z4*Y3)*ZC+Z4*ZS/Z)*Z2+
      *(((1+Z4*Y3)*Z*ZS+Z4*ZC))
      HS3=((Y3*ZC+ZS/Z)*(1+Z2*Y1))+
      *(Z*ZS*Y3*Y1+Y1*ZC))
50     HS4=((Z*Y3*ZC+Z2*ZS/Z))+
      *(Z*ZS*Y3+ZC))
      HS5=1/(HS1+HS2+HS3+HS4)
      F1=CABS(HS5)
      Y(1,1)=20*A LOG10(F1)
      X(1)=W1
55     PRINT 33,W1,Y(1,1)
      33      FORMAT(15X,'*',2X,F5.2,4X,'*',5X,F14.8,4X,'*')
      CONTINUE
10     PRINT 27
      READ*,(A(I),I=1,160)
      19      FORMAT(80A1)
      CALL USPLH(X,Y,201,1,1,201,A,IMAG4,IER)
      STOP
      END

```

PROGRAM HPN4

73/174 OPT=1

FTN 4.8+498

80/03/05.

PROGRAM HPN6

73/174 OPT=1

FTN 4.8+498

80/03/05

```

1      PROGRAM HPN6(INPUT,OUTPUT)
      C   THIS PROGRAM GIVES THE HP FREQUENCY RESPONSE OF
      C   MIXED LUMPED-DISTRIBUTED STRUCTURE WITH SIX LUMPED
5      C   ELEMENTS. THE CASCADED NETWORK IS DOUBLY TERMINATED
      C   IN 1 OHM RESISTOR.
      C   THE TRANSFER FUNCTION OF THE NETWORK IS FOUND TO
      C   BE,  $TRFN = 1/(A+B+C+D)$ . THEREFORE HS1, HS2, HS3, HS4
10     C   REPRESENT THE CHAIN PARAMETERS A, B, C, D OF THE
      C   NETWORK IN THAT ORDER.
      C   COMPLEX S, HS1, HS2, HS3, HS4, HS5, Z8, ZC, Y1, Z2, Y3, Z4, Y5, Z6
      C   DIMENSION X(201), Y(201,1), A(160), IMAG4(5151)
15     REAL C1, L2, C3, L4, C5, L6, D, Z
      READ*, C1, L2, C3, L4, C5, L6, D, Z
      PRINT 11
      PRINT*, "ELEMENT VALUES"
      PRINT*, "-----"
20     11  FORMAT(1H1,5(/))
      PRINT*, "L1= ", C1, " L2= ", L2, " C3= ", C3, " L4= ", L4
      PRINT*, "C5= ", C5, " L6= ", L6, " D= ", D, " Z= ", Z
      PRINT 17
25     17  FORMAT(3X,40(1H,)/)
      PRINT 27
      PRINT 22
      22  FORMAT(15X,"*", " FREQ(H/8)" ,3X,"*",4X,"ATTENUATION (DB)", " *")
      PRINT 27
      27  FORMAT(15X,38(1H,))
30     DO 10 I=1,201,5
      W1=(I)*0.01
      W=1/W1
      S=CMPLX(0.0,W)
      ZS=CMPLX(0.0,SIN(W*D))
35     ZC=CMPLX(COS(W*D),0.0)
      Y1=C1*S
      Z2=L2*S
      Y3=C3*S
      Z4=L4*S
40     Y5=C5*S
      Z6=L6*S
      HS1=((1+Z6*Y5)*ZC+((1+Z6*Y5)*Z4+Z6)*ZS/Z+
45     S*((1+Z6*Y5)*Z*ZS+((1+Z6*Y5)*Z4+Z6)*ZC)*Y3
      S*((1+Z6*Y5)*ZC+((1+Z6*Y5)*Z4+Z6)*ZS/Z+
      S*((1+Z6*Y5)*Z*ZS+((1+Z6*Y5)*Z4+Z6)*ZC)*Y3
      S)*Z2+((1+Z6*Y5)*Z*ZS+((1+Z6*Y5)*Z4+Z6)*ZC)*Y1
      HS2=((1+Z6*Y5)*ZC+((1+Z6*Y5)*Z4+Z6)*ZS/Z+
50     S*((1+Z6*Y5)*Z*ZS+((1+Z6*Y5)*Z4+Z6)*ZC)*Y3)
      S*Z2+((1+Z6*Y5)*Z*ZS+((1+Z6*Y5)*Z4+Z6)*ZC)*ZC
      HS3=Y5*ZC+(1+Y5*Z4)*ZS/Z+(Y5*Z*ZS+(1+Y5*Z4)
55     S*ZC)*Y3+((Y5*ZC+(1+Y5*Z4)*ZS/Z+(Y5*Z*ZS+
      S*(1+Y5*Z4)*ZC)*Y3)*Z2+Y5*Z*ZS+(1+Y5*Z4)*ZC)*Y1
      HS4=(Y5*ZC+(1+Y5*Z4)*ZS/Z+(Y5*Z*ZS+(1+Y5*Z4)*
      S*ZC)*Y3)*Z2+Y5*Z*ZS+(1+Y5*Z4)*ZC
      HS5=1/(HS1+HS2+HS3+HS4)
      F1=CABS(HS5)
      Y(I,1)=20*ALOG10(F1)
      X(I)=W1
45     33  PRINT 33,W1,Y(I,1)
      33  FORMAT(15X,"*",2X,F5.2,4X,"*",5X,F14.8,4X,"*")
      10  CONTINUE
      PRINT 27
      READ 15,(A(I),I=1,160)
70     15  FORMAT(80A1)
      CALL USPLH(X,Y,201,1,1,201,A,IMAG4,IER)
      STOP
      END

```

PROGRAM HPN6

73/174 OPT=1

FTN 4.8+498

80/03/05











MP RESPONSE WITH BUTTERFLY ELEMENT VALUES FOR  $\alpha = 0.2$  AND  $0 = 0.6$

**-6792E01**

-1390E+02

2030E012

2007E002

200396E012

28435482

20034569

28034E08Z

62113E+02

-6792E08

00030209

**●**

10-39821-

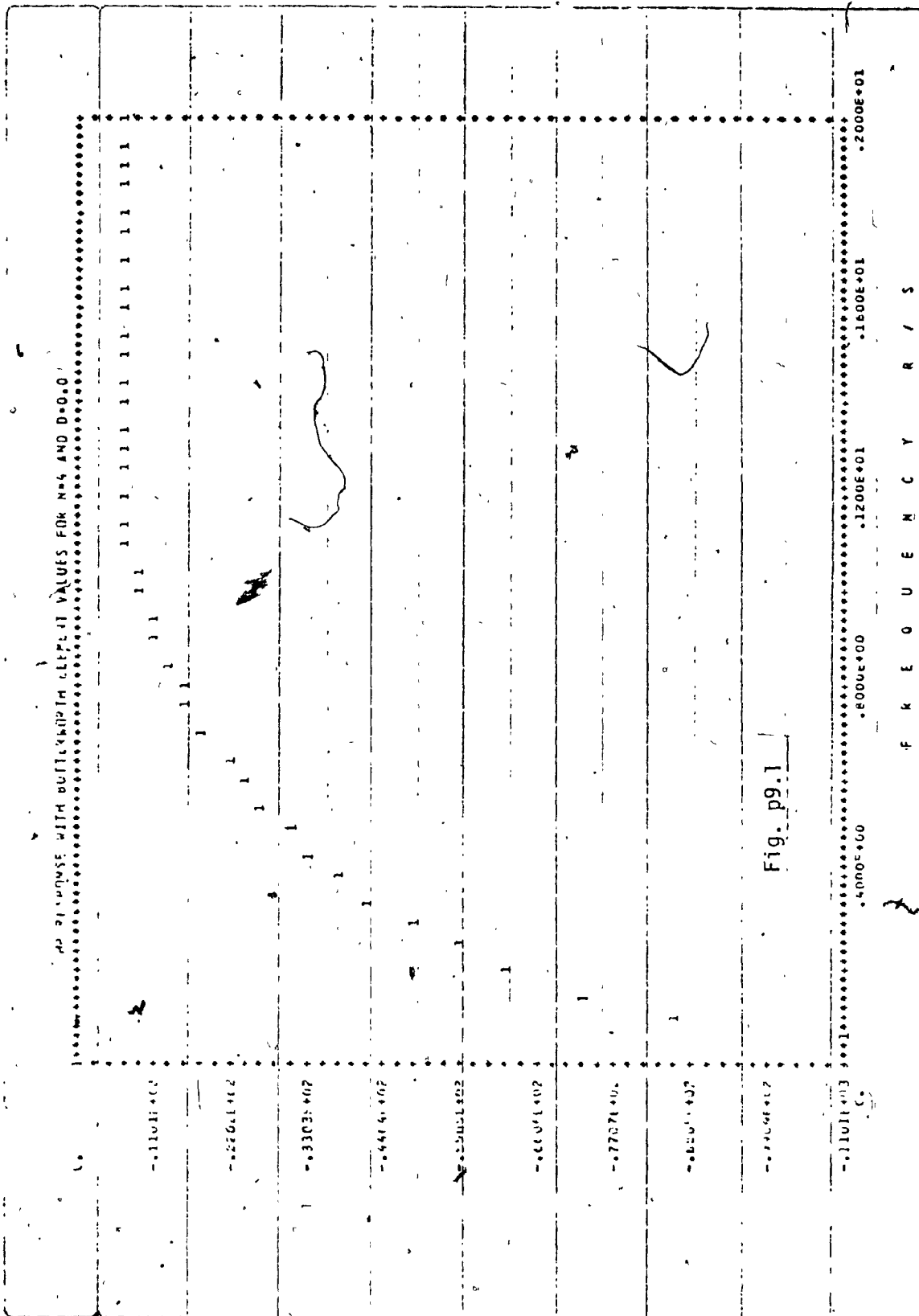
1508E01

100-307027-2010E-01

Fig. p8.5

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JUN 1 1964

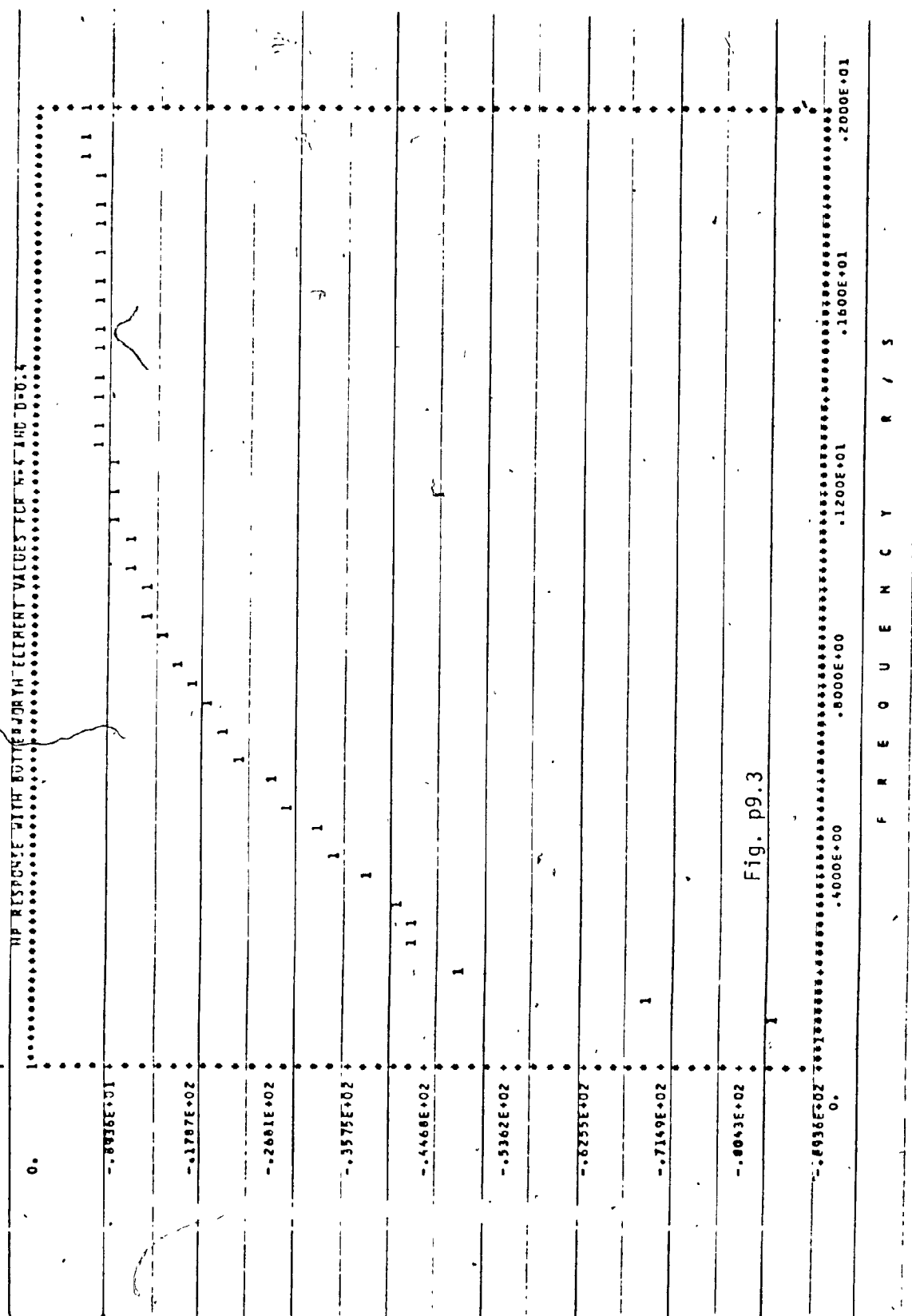




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HF RESPONSE WITH RESPECT TO ELEMENT VALUES FOR N3 AND 0-0-6									
0.	1	.....	.....	.....	.....	.....	.....	.....	.....
-1043E+02	1	.....	.....	.....	.....	.....	.....	.....	.....
-2166E+02	1	.....	.....	.....	.....	.....	.....	.....	.....
-3250E+02	1	.....	.....	.....	.....	.....	.....	.....	.....
-4333E+02	1	.....	.....	.....	.....	.....	.....	.....	.....
-5416E+02	1	.....	.....	.....	.....	.....	.....	.....	.....
-6499E+02	1	.....	.....	.....	.....	.....	.....	.....	.....
-7583E+02	1	.....	.....	.....	.....	.....	.....	.....	.....
-8666E+02	1	.....	.....	.....	.....	.....	.....	.....	.....
-9749E+02	1	.....	.....	.....	.....	.....	.....	.....	.....
-1083E+03	1	.....	.....	.....	.....	.....	.....	.....	.....
0.	1	.....	.....	.....	.....	.....	.....	.....	.....
F R E E U E N C Y / S									
CONCORDIA UNIVERSITY									
COMPUTER CENTRE									

Fig. p9.4



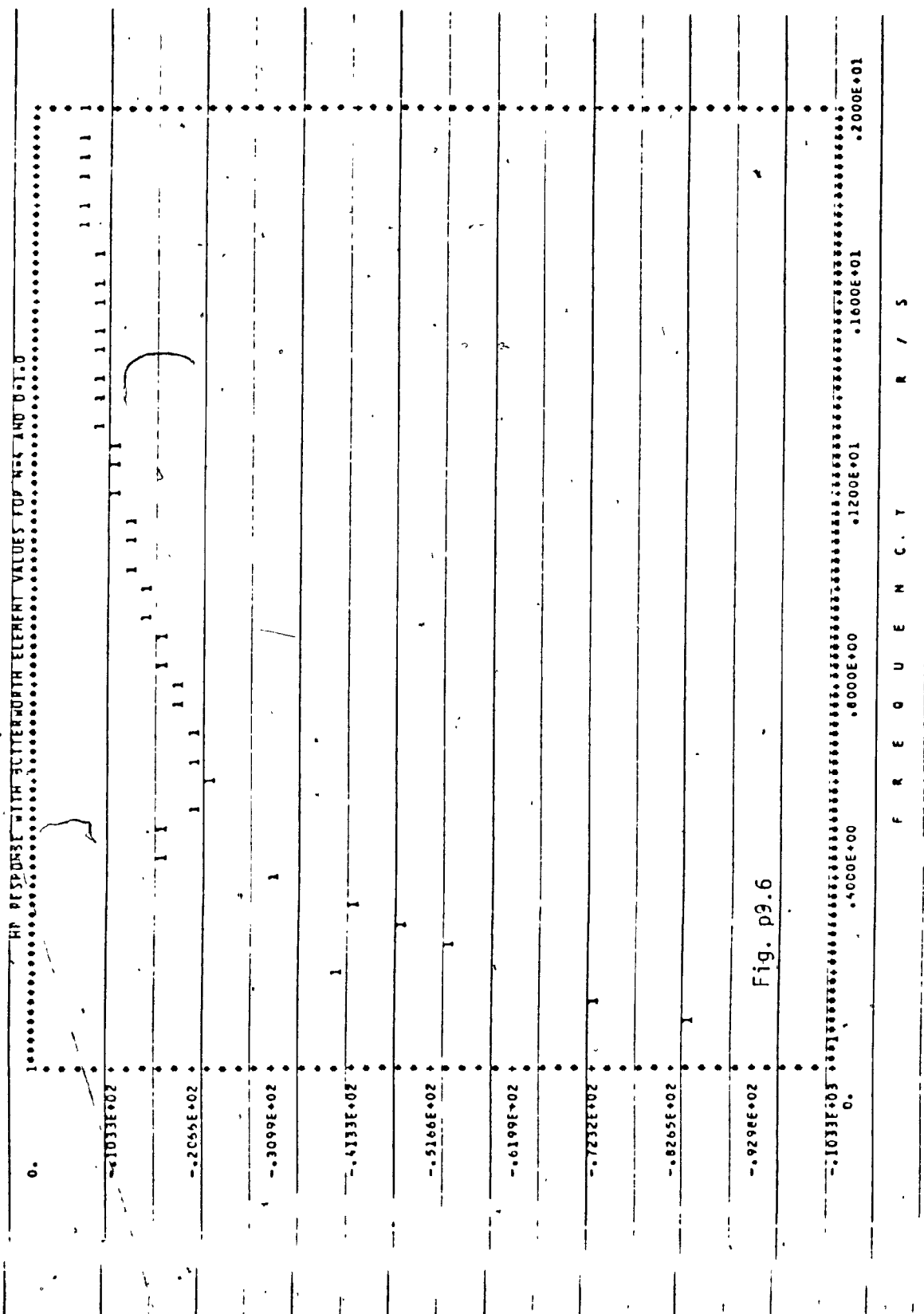


Fig. p9.6

















### HP RESPONSE WITH CMBYSHEV ELEMENT VALUES FOR $N=2$ AND $D=0.2$

9

-7857E+01

-.1571E+02

-02357E+02

3163E02

3929E+12

-0.4714E+02

**21031155-**

123456789

-707ZE-02

Fig. p11.2

7037E+02

2

BB-3828-88

**● 重要事項**

10-39821-1286F-01

7038691-

20180102

3  
7  
8  
A  
C  
H  
E  
N  
O  
3  
8  
3

HP RESPONSE WITH CHEBYSHEV ELEMENT VALUES FOR N=2 AND D=3.4									
0.	1	1	1	1	1	1	1	1	1
-.0241E+01	1	1	1	1	1	1	1	1	1
-.1644E+02	1	1	1	1	1	1	1	1	1
-.2472E+02	1	1	1	1	1	1	1	1	1
-.3296E+02	1	1	1	1	1	1	1	1	1
-.4120E+02	1	1	1	1	1	1	1	1	1
-.4944E+02	1	1	1	1	1	1	1	1	1
-.5768E+02	1	1	1	1	1	1	1	1	1
-.6592E+02	1	1	1	1	1	1	1	1	1
-.7417E+02	1	1	1	1	1	1	1	1	1
-.8241E+02	1	1	1	1	1	1	1	1	1
-.9065E+02	1	1	1	1	1	1	1	1	1
-.9889E+02	1	1	1	1	1	1	1	1	1
-.1071E+03	1	1	1	1	1	1	1	1	1
-.1153E+03	1	1	1	1	1	1	1	1	1
-.1235E+03	1	1	1	1	1	1	1	1	1
-.1317E+03	1	1	1	1	1	1	1	1	1
-.1399E+03	1	1	1	1	1	1	1	1	1
-.1481E+03	1	1	1	1	1	1	1	1	1
-.1563E+03	1	1	1	1	1	1	1	1	1
-.1645E+03	1	1	1	1	1	1	1	1	1
-.1727E+03	1	1	1	1	1	1	1	1	1
-.1809E+03	1	1	1	1	1	1	1	1	1
-.1891E+03	1	1	1	1	1	1	1	1	1
-.1973E+03	1	1	1	1	1	1	1	1	1
-.2055E+03	1	1	1	1	1	1	1	1	1
-.2137E+03	1	1	1	1	1	1	1	1	1
-.2219E+03	1	1	1	1	1	1	1	1	1
-.2301E+03	1	1	1	1	1	1	1	1	1
-.2383E+03	1	1	1	1	1	1	1	1	1
-.2465E+03	1	1	1	1	1	1	1	1	1
-.2547E+03	1	1	1	1	1	1	1	1	1
-.2629E+03	1	1	1	1	1	1	1	1	1
-.2711E+03	1	1	1	1	1	1	1	1	1
-.2793E+03	1	1	1	1	1	1	1	1	1
-.2875E+03	1	1	1	1	1	1	1	1	1
-.2957E+03	1	1	1	1	1	1	1	1	1
-.3039E+03	1	1	1	1	1	1	1	1	1
-.3121E+03	1	1	1	1	1	1	1	1	1
-.3203E+03	1	1	1	1	1	1	1	1	1
-.3285E+03	1	1	1	1	1	1	1	1	1
-.3367E+03	1	1	1	1	1	1	1	1	1
-.3449E+03	1	1	1	1	1	1	1	1	1
-.3531E+03	1	1	1	1	1	1	1	1	1
-.3613E+03	1	1	1	1	1	1	1	1	1
-.3695E+03	1	1	1	1	1	1	1	1	1
-.3777E+03	1	1	1	1	1	1	1	1	1
-.3859E+03	1	1	1	1	1	1	1	1	1
-.3941E+03	1	1	1	1	1	1	1	1	1
-.4023E+03	1	1	1	1	1	1	1	1	1
-.4105E+03	1	1	1	1	1	1	1	1	1
-.4187E+03	1	1	1	1	1	1	1	1	1
-.4269E+03	1	1	1	1	1	1	1	1	1
-.4351E+03	1	1	1	1	1	1	1	1	1
-.4433E+03	1	1	1	1	1	1	1	1	1
-.4515E+03	1	1	1	1	1	1	1	1	1
-.4597E+03	1	1	1	1	1	1	1	1	1
-.4679E+03	1	1	1	1	1	1	1	1	1
-.4761E+03	1	1	1	1	1	1	1	1	1
-.4843E+03	1	1	1	1	1	1	1	1	1
-.4925E+03	1	1	1	1	1	1	1	1	1
-.5007E+03	1	1	1	1	1	1	1	1	1
-.5089E+03	1	1	1	1	1	1	1	1	1
-.5171E+03	1	1	1	1	1	1	1	1	1
-.5253E+03	1	1	1	1	1	1	1	1	1
-.5335E+03	1	1	1	1	1	1	1	1	1
-.5417E+03	1	1	1	1	1	1	1	1	1
-.5499E+03	1	1	1	1	1	1	1	1	1
-.5581E+03	1	1	1	1	1	1	1	1	1
-.5663E+03	1	1	1	1	1	1	1	1	1
-.5745E+03	1	1	1	1	1	1	1	1	1
-.5827E+03	1	1	1	1	1	1	1	1	1
-.5909E+03	1	1	1	1	1	1	1	1	1
-.5991E+03	1	1	1	1	1	1	1	1	1
-.6073E+03	1	1	1	1	1	1	1	1	1
-.6155E+03	1	1	1	1	1	1	1	1	1
-.6237E+03	1	1	1	1	1	1	1	1	1
-.6319E+03	1	1	1	1	1	1	1	1	1
-.6401E+03	1	1	1	1	1	1	1	1	1
-.6483E+03	1	1	1	1	1	1	1	1	1
-.6565E+03	1	1	1	1	1	1	1	1	1
-.6647E+03	1	1	1	1	1	1	1	1	1
-.6729E+03	1	1	1	1	1	1	1	1	1
-.6811E+03	1	1	1	1	1	1	1	1	1
-.6893E+03	1	1	1	1	1	1	1	1	1
-.6975E+03	1	1	1	1	1	1	1	1	1
-.7057E+03	1	1	1	1	1	1	1	1	1
-.7139E+03	1	1	1	1	1	1	1	1	1
-.7221E+03	1	1	1	1	1	1	1	1	1
-.7303E+03	1	1	1	1	1	1	1	1	1
-.7385E+03	1	1	1	1	1	1	1	1	1
-.7467E+03	1	1	1	1	1	1	1	1	1
-.7549E+03	1	1	1	1	1	1	1	1	1
-.7631E+03	1	1	1	1	1	1	1	1	1
-.7713E+03	1	1	1	1	1	1	1	1	1
-.7795E+03	1	1	1	1	1	1	1	1	1
-.7877E+03	1	1	1	1	1	1	1	1	1
-.7959E+03	1	1	1	1	1	1	1	1	1
-.8041E+03	1	1	1	1	1	1	1	1	1
-.8123E+03	1	1	1	1	1	1	1	1	1
-.8205E+03	1	1	1	1	1	1	1	1	1
-.8287E+03	1	1	1	1	1	1	1	1	1
-.8369E+03	1	1	1	1	1	1	1	1	1
-.8451E+03	1	1	1	1	1	1	1	1	1
-.8533E+03	1	1	1	1	1	1	1	1	1
-.8615E+03	1	1	1	1	1	1	1	1	1
-.8697E+03	1	1	1	1	1	1	1	1	1
-.8779E+03	1	1	1	1	1	1	1	1	1
-.8861E+03	1	1	1	1	1	1	1	1	1
-.8943E+03	1	1	1	1	1	1	1	1	1
-.9025E+03	1	1	1	1	1	1	1	1	1
-.9107E+03	1	1	1	1	1	1	1	1	1
-.9189E+03	1	1	1	1	1	1	1	1	1
-.9271E+03	1	1	1	1	1	1	1	1	1
-.9353E+03	1	1	1	1	1	1	1	1	1
-.9435E+03	1	1	1	1	1	1	1	1	1
-.9517E+03	1	1	1	1	1	1	1	1	1
-.9599E+03	1	1	1	1	1	1	1	1	1
-.9681E+03	1	1	1	1	1	1	1	1	1
-.9763E+03	1	1	1	1	1	1	1	1	1
-.9845E+03	1	1	1	1	1	1	1	1	1
-.9927E+03	1	1	1	1	1	1	1	1	1
-.10049E+04	1	1	1	1	1	1	1	1	1

Fig. p11.3

F R E Q U E N C Y R / S

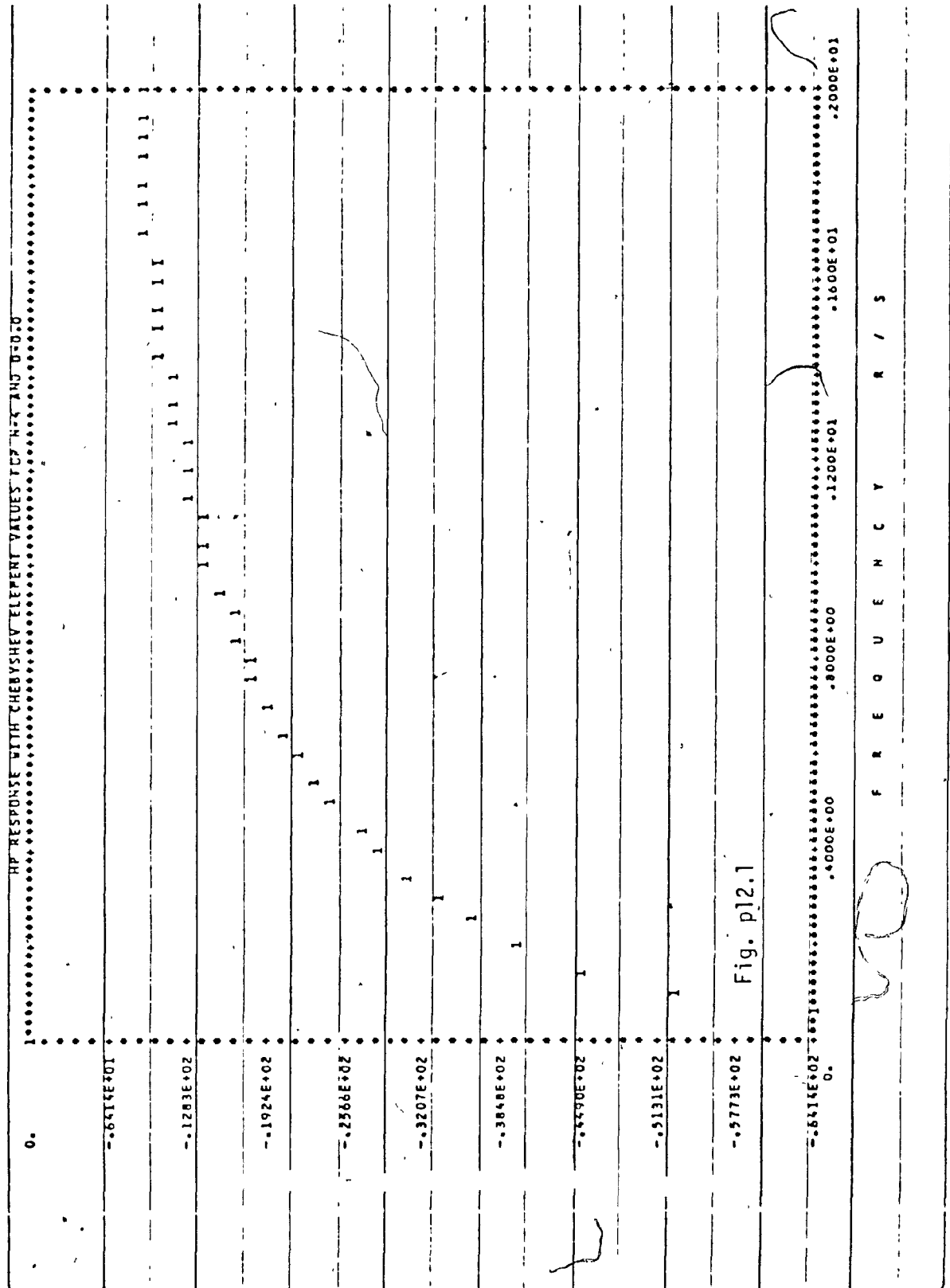


[illegible]

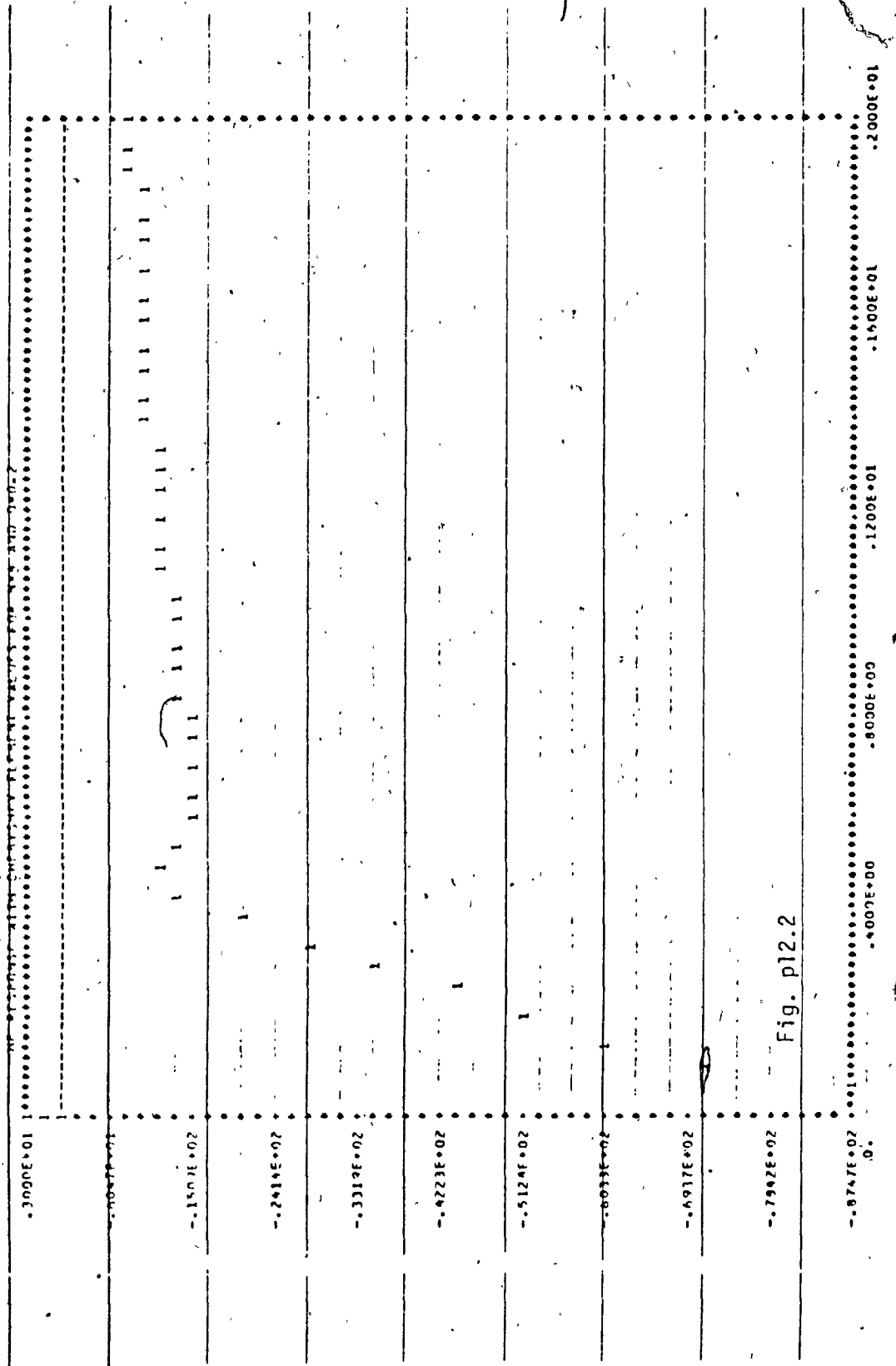
Fig. p11.5

RECEIVED  
Y R / S









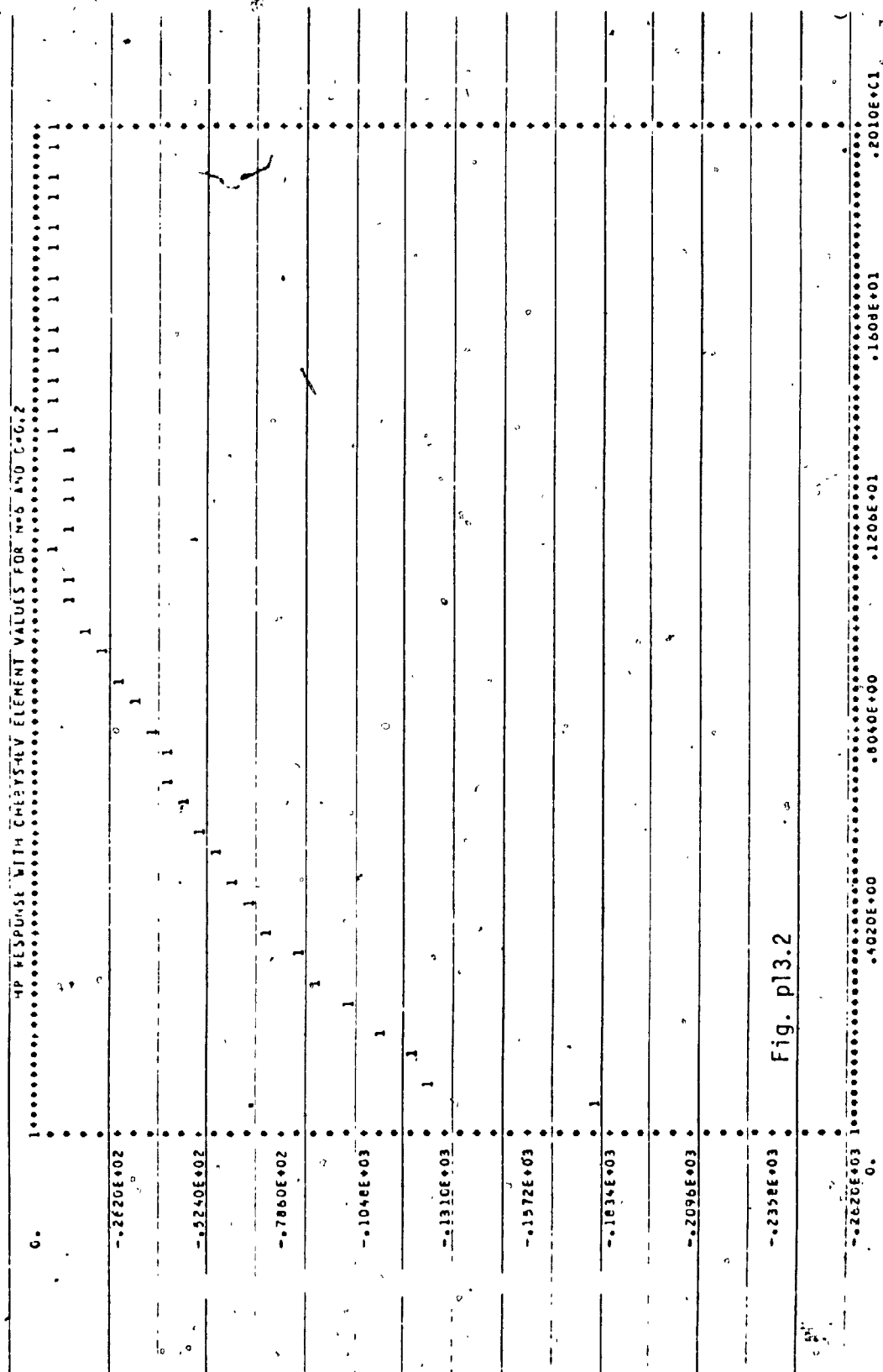












IP RESPONSE WITH CHEBYSHEV ELEMENT VALUES FOR N=6 AND D=0.4									
0.	1	1	1	1	1	1	1	1	1
-.2639E+02	1	1	1	1	1	1	1	1	1
-.5319E+02	1	1	1	1	1	1	1	1	1
-.7978E+02	1	1	1	1	1	1	1	1	1
-.1064E+03	1	1	1	1	1	1	1	1	1
-.1330E+03	1	1	1	1	1	1	1	1	1
-.1596E+03	1	1	1	1	1	1	1	1	1
-.1861E+03	1	1	1	1	1	1	1	1	1
-.2127E+03	1	1	1	1	1	1	1	1	1
-.2393E+03	1	1	1	1	1	1	1	1	1
-.2656E+03	1	1	1	1	1	1	1	1	1
0.	1	1	1	1	1	1	1	1	1
-.4620E+03	1	1	1	1	1	1	1	1	1
-.8040E+00	1	1	1	1	1	1	1	1	1
.1206E+01	1	1	1	1	1	1	1	1	1
.1606E+01	1	1	1	1	1	1	1	1	1
.2010E+01	1	1	1	1	1	1	1	1	1

Fig. p13.3

F R E Q U E N C Y R / S





HP RESPONSE WITH CHEGYSHEV ELEMENT VALUES FOR N=6 AND D=0.8

9

20-36152-1

20+35205.-

02036570

.10CSE.03

1250E+03

-1508E403

1798-03

2016F03 4

22626403

25935.03 10

3

4020E+00

00430708

130450

• • • • •

4

F R E D U E N C Y

Fig. p13.5



The magnitude of attenuation is 64 db's. The increase in the value of 'D' to 0.2 and higher makes the magnitude of attenuation to move into 80's. For values of 'D' greater than 0.4 we have some variations in the magnitude of attenuation near the 3 db frequency which can be termed as the passband ripple.

As seen in the figures p13.1 to 13.4, which is high-pass response for  $n = 6$  with Chebyshev element values, we were unable to obtain a Chebyshev response in this case. The magnitude of attenuation is very high for all values of 'D' and it is above 250 db's. This response can not be termed as a flat response as we do have some increase and decrease in attenuation in the passband but at the same time we can not say that a passband ripple exists.

### 3.3.1 HIGH-PASS BUTTERWORTH FILTER DESIGN

The cutoff frequency for Butterworth filters has to be 1 r/s. In high-pass responses we get this 3 db frequency  $\omega_p$  when the time delay is zero for all the three cases of two, four and six lumped elements with transmission line. Since the cutoff frequency is 1 r/s we can use these as Butterworth filters for different orders.

If we require some specific value of the time delay for any order of the network we can choose the corresponding response and normalize the cutoff frequency to 1 r/s, a procedure similar to the low-pass Butterworth filter design procedures discussed in chapter 2.

We can not design a Butterworth filter for the cases where there is kinks (overshoot) in the response. Butterworth design is only possible if the response is maximally flat.

### 3.3.2 HIGH-PASS CHEBYSHEV FILTER DESIGN

As evident from the responses of figures p11.1 to p11.3, it is not possible to design a Chebyshev filter with two lumped elements as there is no passband ripple in the responses of figures p11.1 to p11.3, which is response for  $n = 2$  with different values of time delay  $D$ .

For  $n = 4$ , we could not get passband ripple for any value of  $D$  as apparent from figure p12.1 to figure p12.5. Thus it is not possible to design a Chebyshev filter in this case.

For  $n = 6$ , we do get some variations in the passband attenuation, but they can not be termed as ripple. Thus it is not possible to design Chebyshev filter with six lumped elements.

## CHAPTER 4

### SUMMARY AND CONCLUSION

#### 4.1 SUMMARY

In this report we have investigated the behavior of the mixed lumped-distributed filters. A uniform lossless transmission line is introduced between the lumped structures of order 1, 2 and 3 as shown in Fig. 1.2.1.2, 1.2.2 and 1.2.3. For these structures we have investigated the Butterworth and Chebyshev type responses both in low-pass and high-pass.

Firstly, the properties of the structure shown in Fig. 1.2.3 are studied. The low-pass response for the Butterworth case was found to have a 3 db response point  $\omega_p$  of 1 r/s. The value of the frequency  $\omega_p$  tends to drop as the time delay is increased. For the time delay of unity  $\omega_p$  is found to be 0.7 r/s. No Chebyshev type response is available for this case.

However Chebyshev type response has been achieved for higher order structures of Fig. 1.2.1.2 and 1.2.2. For the structure of Fig. 1.2.1.2, Chebyshev type response has been obtained for values of time delay higher than or equal to 0.4. The passband ripple varied between 2.0 and 5 db, whereas the  $\omega_p$  varied between 1.8 r/s to 1.57 r/s. The passband ripple and the  $\omega_p$  gradually decreased by increasing the value of time delay. For Butterworth case of this structure, the  $\omega_p$  was found to be 1 r/s for

time delay of zero. The value of  $\omega_r$  dropped to 0.74 r/s for time delay of unity.

For the minimization of passband ripple in Chebyshev case, the computer program 'AFTAB' has been developed. For the desired value of the ripple, the corresponding parameter values are obtained by this program. This program, as we have shown, has reduced the ripple of above 4 db to less than 1 db, but not necessarily equi-ripple. We are even able to obtain maximally flat response as shown in Fig. p3.4. In addition, the elemental values of the lumped portion of the filter is also obtained from this program.

In general we are able to design a mixed lumped-distributed low-pass Butterworth and Chebyshev filters. This kind of filter is better than the lumped filters because of the advantages of transmission line. We are also able to design some high-pass Butterworth filters, however the design of Chebyshev high-pass filters is more difficult than the low-pass case.

#### 4.2 FURTHER POSSIBLE INVESTIGATIONS

The study of the behavior of mixed lumped-distributed structure in high-pass leads to the inference that design of bandpass filters having the structures discussed appears to be more difficult and needs considerable effort and study. This could form one of the projects of future investigation.

The program AFTAB which gives us the value of all the network parameter and the passband ripple, does not give us control over the

frequency  $\omega_r$ .

However further investigation is possible over this problem and it is possible that the user can have a control over this frequency along with the passband ripple.

Investigations can be carried out to obtain equi-ripple passband. This may be highly involved, the reason being that we are trying to obtain three things at a time, the equi-ripple passband within required limits, the desired  $\omega_r$  and the element values.



## REFERENCES

1. "The Real Part of a Multivariable Positive Real Function and The Synthesis of Multivariable Ladder Networks"  
Doctoral thesis by A. Srinavaso Rao  
Concordia University Montreal 1973.
2. "Introduction to Modern Network Synthesis"  
M. E. Valkenberg (Chapters 8, 13, 15)  
John Wiley and Sons,
3. "Network Analysis and Synthesis"  
Louis Weinberg (Chapter 13)  
McGraw-Hill.
4. "Simplified Modern Filter Design"  
Phillip R. Greffe (Chapter 1, 2)  
John F. Rider Publisher Inc.
5. "Computer-Aided Study of a Mixed Lumped-Distributed Filter"  
Major Technical report by Amir N. Gundi  
Concordia University Montreal 1978.

## APPENDIX

IMSL ROUTINE NAME - ZXMIN

PURPOSE - MINIMUM OF A FUNCTION OF N VARIABLES USING A QUASI-NEWTON METHOD

USAGE - CALL ZXMIN (FUNCT,N,NSIG,MAXFN,IOPT,X,H,G,F,W,IER)

ARGUMENTS

FUNCT	- A USER SUPPLIED SUBROUTINE WHICH CALCULATES THE FUNCTION F FOR GIVEN PARAMETER VALUES $X(1), X(2), \dots, X(N)$ . THE CALLING SEQUENCE HAS THE FOLLOWING FORM CALL FUNCT(N,X,F) WHERE X IS A VECTOR OF LENGTH N. FUNCT MUST APPEAR IN AN EXTERNAL STATEMENT IN THE CALLING PROGRAM. FUNCT MUST NOT ALTER THE VALUES OF $X(I), I=1, \dots, N$ OR N.
N	- THE NUMBER OF PARAMETERS (I.E., THE LENGTH OF X) (INPUT)
NSIG	- CONVERGENCE CRITERION. (INPUT). THE NUMBER OF DIGITS OF ACCURACY REQUIRED IN THE PARAMETER ESTIMATES. THIS CONVERGENCE CONDITION IS SATISFIED IF ON TWO SUCCESSIVE ITERATIONS, THE PARAMETER ESTIMATES (I.E., $X(I), I=1, \dots, N$ ) AGREE, COMPONENT BY COMPONENT, TO NSIG DIGITS.
MAXFN	- MAXIMUM NUMBER OF FUNCTION EVALUATIONS (I.E., CALLS TO SUBROUTINE FUNCT) ALLOWED. (INPUT)
IOPT	- OPTIONS SELECTOR. (INPUT) IOPT = 0 CAUSES ZXMIN TO INITIALIZE THE HESSIAN MATRIX H TO THE IDENTITY MATRIX. IOPT = 1 INDICATES THAT H HAS BEEN INITIALIZED BY THE USER TO A POSITIVE DEFINITE MATRIX.
X	- VECTOR OF LENGTH N CONTAINING PARAMETER VALUES. ON INPUT, X MUST CONTAIN THE INITIAL PARAMETER ESTIMATES. ON OUTPUT, X CONTAINS THE FINAL PARAMETER ESTIMATES AS DETERMINED BY ZXMIN.
H	- VECTOR OF LENGTH $N*(N+1)/2$ CONTAINING AN ESTIMATE OF THE HESSIAN MATRIX $D**2F/(DX(I)DX(J)), I, J=1, \dots, N$ . H IS STORED IN SYMMETRIC STORAGE MODE. ON INPUT, IF IOPT = 0, ZXMIN INITIALIZES H TO THE IDENTITY MATRIX. AN INITIAL SETTING OF H BY THE USER IS INDICATED BY IOPT=1. H MUST BE POSITIVE DEFINITE. IF IT IS NOT, A TERMINAL ERROR OCCURS. ON OUTPUT, H CONTAINS AN ESTIMATE OF THE HESSIAN AT THE FINAL PARAMETER ESTIMATES (I.E., AT $X(1), X(2), \dots, X(N)$ )
G	- A VECTOR OF LENGTH N CONTAINING AN ESTIMATE OF THE GRADIENT $DF/DX(I), I=1, \dots, N$ AT THE FINAL PARAMETER ESTIMATES. (OUTPUT)
F	- A SCALAR CONTAINING THE VALUE OF THE FUNCTION AT THE FINAL PARAMETER ESTIMATES. (OUTPUT)

ZXMIN-1

W - A VECTOR OF LENGTH 3\*N USED AS WORKING SPACE.  
ON OUTPUT, WORK(I), CONTAINS FOR  
I = 1, THE NORM OF THE GRADIENT (I.E.,  
SQRT(G(1)\*\*2+G(2)\*\*2+...+G(N)\*\*2))  
I = 2, THE NUMBER OF FUNCTION EVALUATIONS  
PERFORMED.  
I = 3, AN ESTIMATE OF THE NUMBER OF  
SIGNIFICANT DIGITS IN THE FINAL  
PARAMETER ESTIMATES.

IER - ERROR PARAMETER (OUTPUT)  
IER = 0 IMPLIES THAT CONVERGENCE WAS  
ACHIEVED AND NO ERRORS OCCURRED.  
TERMINAL ERROR  
IER = 129 IMPLIES THAT THE INITIAL HESSIAN  
MATRIX IS NOT POSITIVE DEFINITE. THIS  
CAN OCCUR ONLY FOR IOPT = 1.  
IER = 130 IMPLIES THAT THE ITERATION WAS  
TERMINATED DUE TO ROUNDING ERRORS  
BECOMING DOMINANT. THE PARAMETER  
ESTIMATES HAVE NOT BEEN DETERMINED TO  
NSIG DIGITS.  
IER = 131 IMPLIES THAT THE ITERATION WAS  
TERMINATED BECAUSE MAXFN WAS EXCEEDED.

PRECISION/HARDWARE - SINGLE AND DOUBLE/H32  
- SINGLE/H36, H48, H60

REQD. IMSL ROUTINES - UERTST, UGETIO, ZXMIN

NOTATION - INFORMATION ON SPECIAL NOTATION AND  
CONVENTIONS IS AVAILABLE IN THE MANUAL  
INTRODUCTION OR THROUGH IMSL ROUTINE UHELP

#### Algorithm

ZXMIN is based on the Harwell library routine VAL0A. It uses a quasi-Newton method to find the minimum of a function  $f(x)$  of  $N$  variables  $x = (x_1, x_2, \dots, x_N)$ .

See reference:

Fletcher, R., "Fortran subroutines for minimization by quasi-Newton methods", Report R7125 AERE, Harwell, England, June, 1972.

#### Programming Notes

1. It is assumed that the gradient vector  $g(x) = (\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_N})$  and the Hessian matrix  $H = (\frac{\partial^2 f}{\partial x_i \partial x_j})$  exist, but the user is not required to supply formulas for their evaluation.
2. Accuracy in evaluating the function  $f$  is critical when highly accurate parameter values,  $x_j$ , are required (i.e., when NSIG is greater than 3 for single precision). In these cases, it is advisable to evaluate  $f$  in precision greater than working precision (e.g., for single precision ZXMIN, double precision can be used).

ZXMIN-2

3. In reference to significant digit tests for the  $x$ -vector, leading zeroes to the right of the decimal point are counted. For example, both 123.45 and 0.00123 have five significant digits. Scaling the  $x$  vector may be required if several of the parameters  $x_j$  are much less than 1.0 to obtain the same number of significant digits for each of the  $x_j$ .
4. The user can place a limit on the amount of computer time used by ZXMIN by setting MAXFN appropriately. In general, MAXFN=500 is satisfactory with NSIG=3.
5. The accuracy of the final Hessian matrix,  $H$ , is not always satisfactory. For example, when a minimum is located in a very few steps, the estimate may be inaccurate.  $H$  is calculated by updating the initial Hessian as steps occur.